Teaching Mathematics Today

2nd Edition

Table of Contents

- Teachers can read the extensive explanations of the mathematics teaching strategies and integrate them into their own lessons.
- Teachers might use this book to improve the effectiveness of mathematics instruction in their classroom.
- A school, mathematics department, or PLC might work through the entire book when streamlining and improving a mathematics program.
- Mentors and coaches can use this book as a guide as they work with teachers, and they can read this book in a study group to discuss best practices in mathematics teaching.
- New and veteran teachers can read and apply the techniques described in the chapters to their current instruction of mathematical concepts.
- Teachers can deepen their knowledge of differentiation to meet the different needs of, and offer access to core curriculum to struggling students and English language learners.

Ultimately, the suggestions in this book will assist teachers in developing their toolbox of research-based teaching strategies to address different learning styles, engage students, and differentiate instruction.

How This Book Is Organized

This book is designed to be read in sequential order. Each chapter focuses on a different aspect of teaching in a mathematics classroom. Here's a brief summary of the topics covered:

- **Chapter 1—Understanding the Role of the Teacher in the Mathematics Classroom:** It is imperative for teachers to develop character trust and competency trust with their students in order to facilitate learning. Students need to wrestle with mathematical concepts in a "productive struggle"and have opportunities to explain and justify their thinking. This chapter provides multiple questioning strategies teachers can use immediately. Because teachers can no longer afford

to work in isolation, this chapter also covers collaborative teams of teachers whose members work interdependently to achieve common goals. PLC members are mutually accountable and realize that all of their efforts must be assessed on the basis of results rather than intentions (DuFour et al. 2010).

- **Chapter 2—Managing a Successful Mathematics Classroom:** When strong classroom management techniques are implemented, students have more opportunities for learning. It is important for teachers to have routines and procedures and review them frequently with students. This chapter provides multiple resources for mathematics teachers as they plan for and develop their own classroom management strategies. The planning tools in this chapter will help guide teachers as they implement a workshop model, allowing students to explore and be engaged in mathematics and establishing a sense of community.
- **Chapter 3—Planning for Instruction:** When planning for instruction, it is crucial for mathematics teachers to unpack standards for their grade level and course. Teachers need to attend to the rigor at which the standard is written and hold students to that level of understanding. This chapter offers resources to use when unpacking standards and writing learning targets for instruction.
- **Chapter 4—Implementing Mathematical Practice and Process Standards:** Mathematical practice standards are the "how" in terms of content proficiency. Teachers need to provide opportunities for students to practice engaging in mathematical dialogue in which they defend, explain, justify, and question each other. This chapter provides tools and strategies to reinforce mathematical practices and incorporate them into the mathematics classroom through cooperative learning. When students are actively motivated and busy reaching learning goals, they are also constructing knowledge and moving toward successful mastery of the mathematical content standards.
- **Chapter 5—Building Conceptual Understanding:** Research shows that students who are not successfully mastering mathematical concepts tend to demonstrate slow or inaccurate retrieval of basic mathematical facts, lean toward impulsivity when solving problems, and have difficulty forming mental representations of mathematical concepts or keeping information in working memory (Gersten and Clarke 2007). This chapter offers a step-by-step process to teach students problem-solving strategies along with various vocabulary

development activities. The set activity examples are meant to develop independent, competent student problem-solvers.

- **Chapter 6—Assessing Students:** To reach the goal of all students mastering the given curriculum with appropriate instruction, materials, and support, mathematics teachers must use summative and formative assessment strategies "minute by minute and day by day, to adjust their instruction to meet their students' learning needs" (Wiliam 2007). Assessments provide teachers with the necessary data to understand which students are struggling in specific areas of the curriculum and which students need enrichment. This chapter provides strategies, charts, and rubrics for summative and formative assessments, as well as ways to use data to further drive instruction in the classroom.
- **Chapter 7—Supporting Instruction Through Differentiation:** Students should have multiple experiences with topics, allowing them to integrate the topics into their knowledge base (Marzano 2003). However, not all students process the new information in the same way or bring the same skill set to the learning experience. Some students need extra time to process concepts and look at problems in different ways (Sutton and Krueger 2002). Other students may require intense intervention—further teaching or material presented in multiple ways. This chapter provides charts, strategies, and tips for identifying individual student needs and ways to differentiate mathematics instruction to meet those needs.
- **Chapter 8—Integrating Mathematics Across the Curriculum:** Rather than working on subjects in isolation from one another—studying reading apart from writing and apart from math, science, social studies, and other curricular areas—children learn best when they are engaged in inquiries that involve using language to learn and that naturally incorporate content from a variety of subject areas (NCTE 1993). It is important for students to understand that education is not a series of compartmentalized subjects that have nothing to do with one another. Rather, students need to realize that learning is more like a tapestry, where all subjects are woven together to create a broad scope of understanding that is ultimately most useful when all the strands fit together. This chapter provides suggestions and strategies for teachers to integrate mathematics across the curriculum. It includes information about reaching all learners and broadening students' understanding of mathematical concepts.

- **Appendix A:** Teaching mathematics in today's diverse classrooms can be challenging, but it also provides teachers with many exciting opportunities to pass on life skills as well as mathematical knowledge. This section offers general information and suggestions for all teachers and provides additional tips and advice for new teachers.

Conclusion

This book is intended to help teachers navigate today's mathematics classroom. Closed doors are a thing of the past; conversations with fellow educators will help teachers meet the diverse needs of all learners. With a focus on student thinking and learning, this book provides strategies and techniques for teachers to explore and try. Change can only happen through effective teaching by first understanding the role of the teacher, as described next.

Chapter 4

Implementing Mathematical Practice and Process Standards

"Let's face it; by and large math is not easy, but that's what makes it so rewarding when you conquer a problem, and reach new heights of understanding." (Danica McKellar 2014)

Mathematical practice and process standards describe what students need to do and know to master essential mathematics skills and concepts. The Common Core State Standards for Mathematics (CCSSM) Practice Standards, for example, were built on "processes and proficiencies" from the National Council of Teachers of Mathematics (NCTM 2000) process standards of problem solving, reasoning and proof, communication, representation, and connections, along with the strands of mathematical proficiency specified in the National Research Council's report (2001) *Adding It Up: Helping Children Learn Mathematics* on adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

Mathematical practice and process standards are generally introduced in kindergarten and emphasized throughout students' K–12 education. These standards should not be based on specific content, but more broadly on how students will learn the content. Students will continually refine their work as mathematicians, and the standards are the vehicle through which students will continue to learn.

Teachers need to be explicit about which state or district practice and process standards are being emphasized in each lesson. Most of these standards will intertwine throughout the lesson, but teachers should

emphasize or focus on one or two overarching practice and process standards. When planning a lesson, for example, a teacher implementing the CCSSM would use the chart in Figure 4.1 to identify student language when focusing on one of the eight Standards for Mathematical Practice and highlight teacher questions to emphasize the standard in the lesson. The teacher would write down these questions along with pocket questions for quick access. Students can pick one or two sentence stems daily to use when working with their cooperative group until they naturally develop this language.

Figure 4.1 Questions Aligned to Practice Standards

Practice Standard	Student Language	Teacher Questions
1. Make sense of problems and persevere in solving them.	What are we supposed to do? The problem tells us.... Let's make a plan. How can we monitor our progress? Does this make sense? Let's check our answers.	What is the problem asking? What do you need to find? Have you solved problems like this before? How could you start the problem? What manipulatives/tools might help you? Does your plan make sense? How can you check your answer?
2. Reason abstractly and quantitatively.	What do we know? Does this make sense? How can we label this?	How can you represent the problem with symbols or numbers? What do the numbers or variables refer to?
3. Construct viable arguments and critique the reasoning of others.	How did you get your answer? I think _____ because _____. Why does that work? Explain to me your thinking. I disagree, because _____. Show me how you got your answer.	Compare your answer to your partner's answer. How are they the same/different? What does your answer mean? Can you prove your solution to me? Can you find a counterexample? What questions do you have for your partner about this problem?

Practice Standard	Student Language	Teacher Questions
4. Model with mathematics.	What formula will help us? How can we apply what we know? What do we know? Does this make sense?	What connections do you see? How can you write this using words or symbols? Explain the result to me. Is this true for all cases? Can you predict the next one? What assumptions are you making?
5. Use appropriate tools strategically.	What tools could we use? • Calculator • Computer • Model • Paper/pencil • Protractor • Ruler • Spreadsheet	What tool/manipulative could help you with your thinking? Which tool will help you best solve this problem?
6. Attend to precision.	Are we all discussing this problem? How did you calculate that answer? Can we explain the solution? Should we round?	Explain your thinking. What is the problem asking? Does your answer make sense? How do you know your solution is correct? Does your group agree? Why or why not?
7. Look for and make use of structure.	What patterns do we notice? How can we relate this problem to the real world?	What did you notice when you were working through this problem? Why does this happen? What patterns do you see? Will this work all of the time? Why is this important to the problem?
8. Look for and express repeated reasoning.	Does this always happen? How can we relate this problem to everyday life?	Can you make a rule or generalization? Can you think of a shortcut? How could this problem help in solving another problem?

Processes to Enhance Student Interaction

In most professions, people need to interact and work collaboratively to solve problems. This should be no different in a mathematics classroom. A teacher can create structures so students work cooperatively and support one another as they strive toward mastery of learning. Twenty-first-century skills and Mathematical Habits of Mind/Interaction are structures designed to improve mathematical dialogue and thinking.

Twenty-First-Century Skills

The standards push for productive discourse through similar learning skills, and the Partnership for 21st Century Skills (P21) also emphasizes college and career readiness for every student. To succeed in the twenty-first century, all students will need to perform to high standards and acquire mastery of rigorous core subject material (P21 2011). Teachers can focus on the following twenty-first-century learning skills to help students productively work together to solve meaningful mathematics problems:

- Creativity and innovation
- Critical thinking and problem solving
- Communication and collaboration

Figure 4.2 shows what students need for the twenty-first century and how to support those needs.

Developing Mathematical Vocabulary

In mathematics, vocabulary is highly specialized—mathematics terms are not often encountered in everyday life. Therefore, all students need an explicit introduction to and explanation of these vocabulary words in order to apply them to their understanding of mathematical concepts. This task is even more difficult for English language learners, as these vocabulary words are not typically the words that English language learners will learn during their structured English language development (ELD) class period. It is up to the mathematics teacher then to ensure that English language learners study and learn the necessary vocabulary to comprehend mathematical concepts and curriculum.

Furthermore, the different areas of mathematics (e.g., number sense and mathematical reasoning) and the various disciplines (e.g., algebra, trigonometry, geometry, and calculus) have different compilations of specialized vocabulary words. Sometimes the words overlap across mathematical areas and disciplines, but often words are specific to just one mathematical area or discipline. It is vital to understand the vocabulary for a specific discipline in mathematics because this knowledge aids in access to the core curriculum. Students develop a deep understanding of mathematical language only through several approaches that develop mathematical concepts and connections (Dacy, Bramford-Lynch, and Salemi 2013). Teachers want students to demonstrate mastery of concepts, but even more important, teachers want students to think mathematically as lifelong learners. This will be possible if they first achieve understanding of the vocabulary words that explain, describe, justify, and facilitate each of the mathematical concepts.

Mathematics uses technical language. It is helpful if teachers use the technical language as often as possible for consistency and continuity among mathematics classrooms. The following sections give examples of how vocabulary can be developed in a mathematics classroom.

Vocabulary Development for English Language Learners

Mathematics teachers need to explicitly and deliberately teach the academic language needed for students to be successful in doing

mathematics, without interrupting students' reasoning (Kanold et al. 2013). It is important for all students learning mathematics to be familiar with the specialized vocabulary embedded within the practice and application of the concepts. English language learners especially need consistent, structured instruction in learning English. They also need well-planned, sheltered instruction throughout content lessons and effective activities to develop mathematical vocabulary (Dean and Florian 2001).

It is not enough to give English language learners a list of words and have them look up the definitions in dictionaries or textbook glossaries. Students who are struggling with learning a language are not going to find the process easier by simply being given more words to sort through. English language learners need context-embedded lesson activities that acquaint them with the necessary words for comprehension of the content and allow them to practice the use of the words in activities that span listening, speaking, reading, and writing actions. The following example could help acquaint English language learners with the use of the term *symmetry* in relation to geometry in a support class or as a scaffold during the lesson.

Draw the lines of *symmetry* within the given shape:

In a support class, the teacher could take time to review rectangle, line symmetry, and reflection terms. Then, the teacher could have students find the lines of symmetry of different shapes. Students may use a mirror, trace paper, or a MIRA (a tool that has the reflective quality of a mirror) to find the lines. During the support class, the teacher can encourage students to use—and even overuse—correct terminology as they work. In the support class or as scaffolding in the regular mathematics classroom, the teacher can have students reflect, orally or in writing, about the vocabulary and work that they accomplished, and help students see that the "reflections" mirror back their thinking. Connections like these are needed to help English language learners gain specific language.

Mathematics teachers need to be cognizant of the language difficulties students who are learning English may have. Many mathematics teachers believe English instruction is the job of the English teacher; however, the English teacher is not focusing on the specialized mathematical vocabulary and the contexts appropriate to it during English class. It is necessary for the mathematics teacher to offer the scaffolding students need to access mathematical concepts. By knowing the language level of each individual student, as described in the next sections, the teacher can plan appropriate lessons that balance vocabulary development, instruction, modeling, interactive activities, and support.

Types of Language Proficiency

One major concept mathematics teachers need to recognize is the difference between the two types of language proficiency for English language learners. Jim Cummins calls the two types of language Basic Interpersonal Communication Skills (commonly referred to as BICS) and Cognitive Academic Language Proficiency (commonly referred to as CALP) (Crawford 2004). BICS refers to a student's *social language*. Proficiency in social language requires no specific instruction and typically takes as little as three years to acquire. This knowledge can be acquired through media saturation, music, and social situations. Students can easily seem very capable in social language because they need it to survive. For example, a teacher may hear a student chatting with friends and converse with that student before or after class, which may lead the teacher to believe that the student has a firm grasp of the English language. However, that same student might be failing assessments, struggling to keep up with assignments, and unable to write well about mathematical content, indicating that the student lacks CALP.

CALP, or academic language, takes seven or more years to acquire. CALP is proficiency in the language of the content areas and of the classroom. A student who has strong CALP has a command of the use of English within content areas. In mathematics, a student with a strong level of CALP is able to understand key vocabulary, use it in the correct context, and write well about his or her understanding of mathematical concepts and procedures. This level of academic language is not learned easily and intuitively, like BICS. This language proficiency comes only with explicit instruction and planned objectives by the content teachers. That is one reason why

vocabulary development lessons are so important for teachers of English language learners to incorporate into mathematics lessons.

Levels of Language Acquisition

Effective mathematics teachers of English language learners also need to know the levels of language acquisition for each English language learner in the classroom. The appropriate lesson for a student new to English is going to look very different from the appropriate lesson for an English language learner who is close to being considered fluent in the English language.

Many states have official assessments meant to determine the level at which a student is able to use English. These assessments cover the areas of listening, speaking, reading, and writing. Some of the assessments have a separate score for each domain of language and a composite score that combines the overall level at which the student is performing in English.

Figure 5.2 outlines the levels of language acquisition of English language learners and offers suggestions to teachers for how to meet those students' needs.

Figure 5.2 Levels of Language Acquisition and Teacher Use

Level	Description	Teachers Should...
Beginning	These students fall into a wide range of limited English comprehension. They have minimal or limited comprehension with no verbal production. Some beginning students are able to give just one- or two-word responses. Some are beginning to comprehend highly contextualized information and are able to speak in very simple sentences.	Provide a lot of context for mathematical concepts. Use physical movement and visuals to explain mathematical vocabulary. Use sentence frames to help students place mathematical concepts into context. Ask yes/no questions or questions in which the answers are embedded. Always include vocabulary development activities.

Level	Description	Teachers Should...
Early intermediate/ intermediate	These students have good comprehension of information in context. They may exhibit restricted ability to communicate ideas, but they can usually reproduce familiar phrases in simple sentences. As they improve in proficiency, their ability to communicate ideas improves, although they may exhibit errors in production, especially when writing or speaking about highly specialized content.	Provide visuals and context for mathematical concepts. Encourage cooperative and interactive activities in order to make mathematical content comprehensible. Ask questions that require simple sentences with known vocabulary. Elicit simple explanations and summaries. Support writing and reading tasks. Often include vocabulary development activities and the proper ways to communicate using the mathematical vocabulary.
Early advanced/ advanced	These students may appear to be fluent in English, but they often struggle when they have to explain their understanding of an answer or write out the procedures of a concept. They lack the ability to fully communicate higher levels of thinking in content-specific academic language.	Provide structured group discussion of concepts before requiring individual practice and writing about mathematical reasoning. Elicit explanations that analyze and synthesize mathematical information. Model the higher levels of thinking with use of specialized vocabulary. Regularly practice vocabulary development activities, and then take students to the next levels of higher-level thinking using the vocabulary.

Integrating Vocabulary Development into Instruction

Interactive vocabulary development activities should be regularly integrated in mathematics lessons in all classrooms. These types of activities are especially necessary for classrooms with English language learners, students struggling with mathematical concepts, or any students who have not shown mastery of the vocabulary.