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Management

Lessons

Unit I—Numbers and Counting

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Mathematics Vocabulary Research and Practice

Words are labels for key concepts in mathematics. Although learning these words is critical to student success in math, teaching them can be challenging. Asking students to look words up in dictionaries or textbook glossaries and then to memorize definitions provides, at best, a short-term solution. Many mathematical ideas are new to students, and most of the concepts are abstract. Moreover, students may have insufficient background knowledge to learn these new concepts well, let alone to use them in computations.

Over 90 percent of all academic vocabulary (including mathematics) derives from Greek and Latin roots.

In this book, we present a systematic, research-based, and engaging alternative to vocabulary memorization: a roots approach. First, we define our terms: a *root* is an umbrella term for "a word part or pattern that carries meaning." Since the human brain is programmed to detect patterns, a roots approach capitalizes on something that our brains do well. The three kinds of roots are prefixes, bases, and suffixes. Nearly every academic word consists of roots. This book presents 34 Greek and Latin word roots that generate hundreds of words.

Our understanding of a word's meaning begins not with the prefix but, rather, with the "base," because the base is the root that provides the core meaning. The bases that lie at the foundation of mathematical vocabulary are "basic" in that they express such essential and readily comprehensible ideas as numbers and counting, geometry, spatial relationships, and the metric system. This is why we say, "Bases are basic." When a student encounters a long mathematical word, a knowledge of roots enables him or her to "divide and conquer" it. The student is then able to identify the word's basic meaning, which might otherwise be confusing or overwhelming. A roots approach to vocabulary empowers students to look inside a long word and identify its roots that provide the keys to its meaning. Moreover, because roots are used in a great number of academic words, a roots approach is ideal for developing students' vocabulary in the various academic content areas. As students learn these word parts and recognize them as the essential components in specific words from math, their growing verbal skills support their increasing ability to comprehend math principles, as well as to increase and enhance their general vocabulary.

Mathematics Vocabulary Research and Practice *(cont.)*

What Does Research Say About Using a Roots Approach?

The size and depth of students' vocabulary is associated with proficiency in reading comprehension (Baumann et al. 2002; Beck, Perfetti, and McKeown 1982; Kame'enui, Carnine, and Freschi 1982; Stahl and Fairbanks 1986).

Morphological analysis (e.g., via a roots approach) is important because it is generative and allows students to make connections among semantically-related words or word families (Nagy and Scott 2000). Developing morphological awareness is an integral component of word learning (Biemiller and Slonim 2001). In a comprehensive review of 16 studies analyzing the effect of instruction in morphological awareness on literacy achievement, Carlisle (2010) observes that people learn morphemes as they learn language.

Classroom-based studies have demonstrated the effectiveness of teaching word parts and context clues in the primary and intermediate grades (Baumann et al. 2002; Baumann et al. 2005; Biemiller 2005; Carlisle 2000; Kieffer and Lesaux 2007; Mountain 2005; Porter-Collier 2010). Research in content-area vocabulary has demonstrated the effectiveness of teaching Greek and Latin word roots, especially for struggling readers (Harmon, Hedrick, and Wood 2005). Moreover, vocabulary knowledge is associated with higher scores on high-stakes tests like the ACT; students with knowledge of Latin score significantly higher on the SAT than those without such knowledge (ACT 2006; LaFleur 1981).

No single instructional method is sufficient. Teachers need a variety of methods that teach word meanings while also increasing the depth of word knowledge (Blachowicz et al. 2006; Lehr, Osborn, and Hiebert 2004). These methods should aim at fostering:

- **Immersion**

  Students need frequent opportunities to use new words in diverse oral and print contexts in order to learn them thoroughly (Blachowicz and Fisher 2006).

- **Metacognitive and metalinguistic awareness**

  Students must understand and know how to manipulate the structural features of language (Nagy and Scott 2000).

- **Word consciousness**

  Word exploration (e.g., etymology) and word play (e.g., puns, riddles, games) help students develop an awareness of and interest in words (Graves and Watts-Taffe 2002; Lehr, Osborn, and Hiebert 2004).

Mathematics Vocabulary Research and Practice *(cont.)*

Why Teach with a Roots Approach?

Teaching with a roots approach is efficient. Over 60 percent of the words students encounter in their reading have recognizable word parts (Nagy et al. 1989). Moreover, content-area vocabulary is largely of Greek and Latin origin (Harmon, Hedrick, and Wood 2005). Many words from Greek and Latin roots meet the criteria for "tier two" words and are appropriate for instruction (Beck, McKeown, and Kucan 2002).

Root study promotes independent word learning (Carlisle 2010). In addition, roots are multipliers—that is, knowledge of one root can help determine the meaning, pronunciation, and spelling of 10, 20, or more English words. With roots, students learn to make connections among words that are semantically related (Nagy and Scott 2000). Research suggests that the brain is a pattern detector (Cunningham 2004). Roots follow linguistic patterns that help students with the meaning, sound, and spelling of English words. Indeed, Latin and Greek roots have consistent orthographic (spelling) patterns (Rasinski and Padak 2013; Bear et al. 2011).

Many English language learners speak first languages semantically related to Latin. For example, more than 75 percent of the words in Spanish come from Latin (Chandler and Schwartz 1961, 1991). Spanish, Portuguese, French, Catalan, Italian, and Rumanian are all classified as "Romance Languages" because they derive from Latin, the language of ancient Romans. Enhancing this natural linguistic connection inherent in many of these languages can accelerate these students' vocabulary growth (Blachowicz et al. 2006).

Many states are beginning to include a study of roots, including Latin and Greek derivations, in their elementary and middle school literacy standards. Indeed, the Common Core State Standards focus extensively on root-specific standards in the "Reading Foundational Skills" and "Language/Vocabulary Acquisition and Use" sections. According to these standards, attention to roots should begin in kindergarten.

prefix

A root at the beginning of a word. For example, in the word *circumference*, the initial *circum-* is a prefix, meaning "around."

base

The core root, which provides a word with its basic meaning. In the word *circumference*, the base is *fer-*, which means "bear," "go."

suffix

A root that ends a word. In the word *circumference*, the final *-ence* is a suffix, meaning "quality," "state," or "thing."

Note: You can find out more about what prefixes and suffixes do on the Digital Resource CD (filename: functions.pdf).

What Is a Root?

A *root* is a word part that contains meaning (and not merely sound). Roots are vocabulary multipliers—each root taught helps students discover the meaning to multiple words. The three types of roots, depending on their placement within a word, are a prefix, base, and suffix.

Mathematics Vocabulary Research and Practice *(cont.)*

Differentiating Instruction

Some students may need additional support. Others may benefit from additional challenge. These ideas may help you differentiate instruction:

- Use visual aids.
- Ask students to sketch or act out words. Others can guess the depicted words.
- Reduce length of activity.
- Pair students. Encourage them to talk about the roots and the activities.
- Challenge students to create new words that contain the root.
- Talk students through the necessary process to complete an activity. Your aim should be to scaffold students' thinking, not to provide answers.
- Alert other teachers (social studies, etc.) of the roots you are working on with students. Ask them to include them when possible in their own instruction with students.
- Have students keep a personal vocabulary journal in which they list the roots and related words they learn. Encourage students to use their new vocabulary in their oral and written language (e.g., "Use at least one word containing the [*hyper-*] root in your math journal entry today.")
- Put the roots and words derived from the roots on display in the classroom. (You may wish to move some of the displays into the hallway or other sites outside of your classroom.)
- Play word games that involve the roots with your students often. Word lists containing the roots in this book are found on the Digital Resource CD (filename: wordlists.pdf).

Students who need additional challenge can a) look for words containing the featured root in their content-area texts, b) write riddles for others to solve using several words that contain the root, or c) use an online resource to find additional words containing the root (e.g., http://www.onelook.com) or to create word puzzles featuring the root (e.g., http://www.puzzlemaker.com).

Like their peers, English language learners benefit from the focus on meaning using research-based strategies to learn new roots and words. Especially if students' native languages derive from Latin (e.g., Spanish), make comparisons to the native languages whenever possible. (You can look online for resources to assist with this.) When Spanish speakers learn to look for roots within words, they will be able to relate many word roots in English to their counterparts in Spanish. Sharing their knowledge with other classmates will help everyone grow.

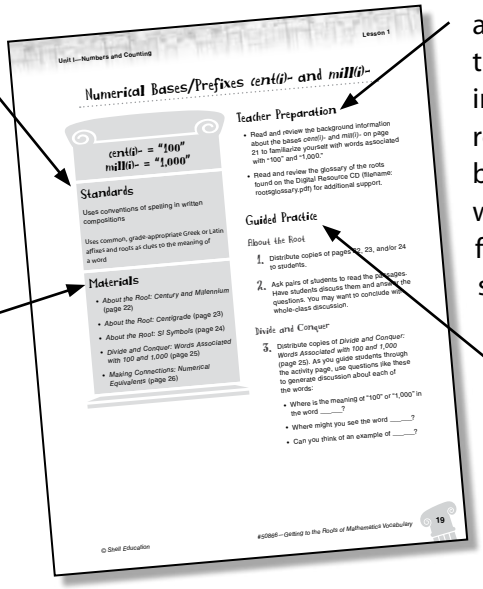
How to Use This Book

The following information will help you implement each lesson.

Lesson Overview

A list of **Standards** (McREL and Common Core State Standards) is included in each lesson.

The **Materials** listed include the activity pages for students.

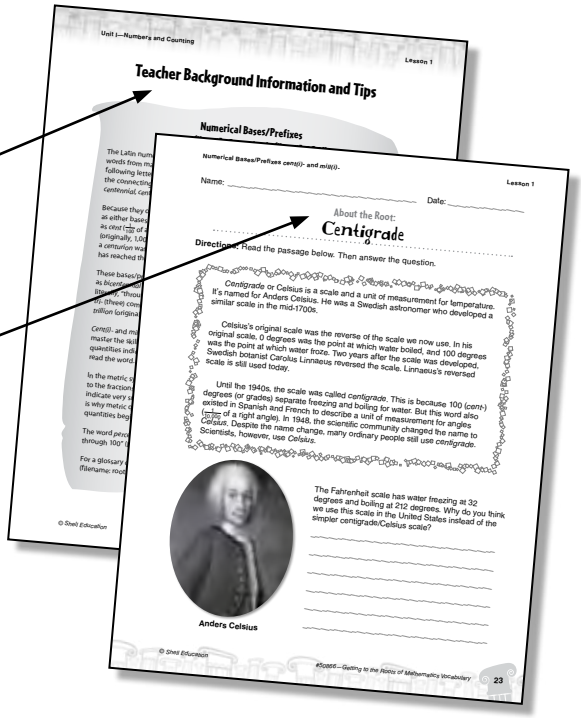


Each lesson begins with a **Teacher Preparation** that provides essential information about the root. Reading this section before you teach the lesson will provide you with a foundation to ensure student success.

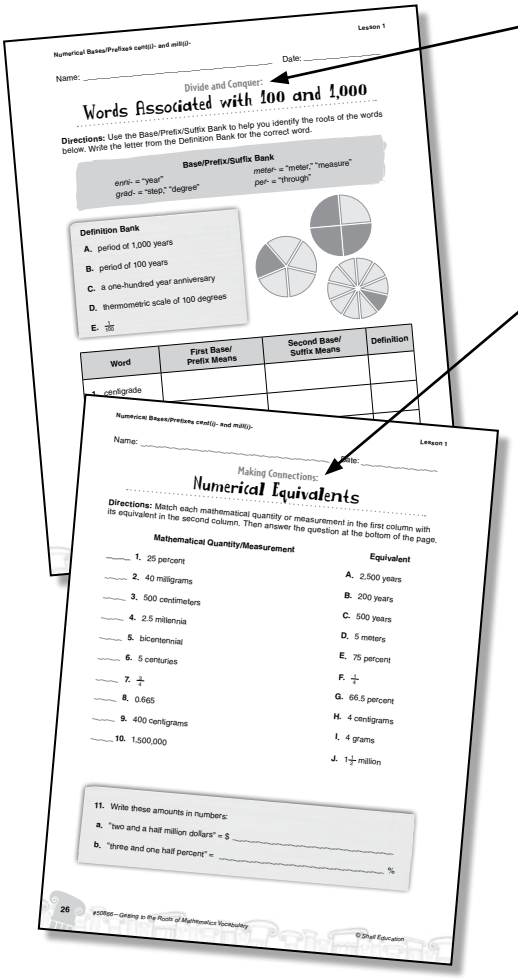
The **Guided Practice** portion of each lesson includes suggestions for implementing each of the student activity pages.

Before beginning each lesson, review the **Teacher Background Information and Tips** page to provide additional help for students. Additional information to introduce each unit can be found on pages 12–13.

The **About the Root** activities are introductions and include short passages using the root of focus. The purpose of these passages is to show students contextual use of the root in mathematics. As students read to themselves or listen to the teacher read aloud, they identify words containing the roots in extended texts that center on a wide range of interesting topics.



How to Use This Book *(cont.)*



Numerical Bases/Prefixes cent(i)- and mill(i)- Lesson 1

Name: _____ Date: _____

Divide and Conquer:

Words Associated with 100 and 1,000

Directions: Use the Base/Prefix/Suffix Bank to help you identify the roots of the words below. Write the letter from the Definition Bank for the correct word.

Base/Prefix/Suffix Bank

enni- = "year"
grad- = "step," "degree"
meter- = "meter," "measure"
per- = "through"

Definition Bank

A. period of 1,000 years
B. period of 100 years
C. a one-hundred year anniversary
D. thermometric scale of 100 degrees
E. $\frac{1}{100}$

Word	First Base/ Prefix Means	Second Base/ Suffix Means	Definition
1. centigrade			

Numerical Bases/Prefixes cent(i)- and mill(i)- Lesson 1

Name: _____ Date: _____

Making Connections:

Numerical Equivalents

Directions: Match each mathematical quantity or measurement in the first column with its equivalent in the second column. Then answer the question at the bottom of the page.

Mathematical Quantity/Measurement	Equivalent
1. 25 percent	A. 2,500 years
2. 40 milligrams	B. 200 years
3. 500 centimeters	C. 500 years
4. 2.5 millennia	D. 5 meters
5. bicentennial	E. 75 percent
6. 5 centuries	F. $\frac{1}{4}$
7. $\frac{3}{4}$	G. 66.5 percent
8. 0.665	H. 4 centigrams
9. 400 centigrams	I. 4 grams
10. 1,500,000	J. $1\frac{1}{2}$ million

11. Write these amounts in numbers:
a. "two and a half million dollars" = $ _____
b. "three and one half percent" = _____ %

26 #50866—Getting to the Roots of Mathematics Vocabulary © Shell Education

The **Divide and Conquer** activities allow students to pull words apart. They dissect the parts of the words, understand the meaning of these parts, and then gain a greater understanding of the word as a whole.

The **Making Connections** activities allow students to use their knowledge of roots to make connections to vocabulary and offer students the opportunity to extend their exploration of the root(s) through activities such as word sorts, riddles, representing the roots and related words in drawings, and game-like tasks. They may need to distinguish when to use a certain root, or which way the root is used in a word.

All of the student activity pages and additional resources such as word lists and flashcards can be found on the **Digital Resource CD**.

How to Use This Book *(cont.)*

Tips for Implementation

These tips will help you think about how to teach the lessons in this book.

- You will find many suggestions in this text, but remember that they are just that—suggestions. You should feel free to adapt the lessons to meet your students' needs.
- Plan to spend five to ten minutes per day on vocabulary related to mathematics.
- You can teach the lessons in any order. You may want to coordinate with your curriculum. Each lesson addresses one basic mathematical idea or concept. You can also expand on any lesson as you see fit. If students need more work on a particular root, you may wish to use some of the additional practice activities described in Appendix C.
- Before beginning a new lesson, read the Teacher Background Information and Tips page. These notes provide general information and identify many math words built on the base of the lesson.
- Talking about roots is very important for student learning. This approach to vocabulary development goes far beyond mere memorization of specific words (which, according to research, does not work). Students need to learn to think about how roots contribute to meanings. Talking this through can help them develop this realization. So, encourage students to talk, Talk, TALK!!!
- Each week, display the root(s) and meaning(s) prominently in your classroom. Encourage students to use the root as much as possible throughout the entire week. Reading, writing, speaking, and listening to words containing the root will facilitate learning. Several generic activities (see Appendix C) provide additional instruction or practice, if you or your students wish.
- You may wish to provide students with dictionaries as they work through the activities sheets.

Bases *gon-* and *angl-, angul-*

***gon-, angl-, angul-* = "angle," "corner"**

Standards

Uses a variety of strategies to extend reading vocabulary

By the end of the year, reads and comprehends literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range

Materials

- *About the Root: Angles and Corners* (page 62)
- *About the Root: Polygons* (page 63)
- *About the Root: Triangles* (page 64)
- *Divide and Conquer: Words Associated with Angle and Corner* (page 65)
- *Making Connections: Riddles* (page 66)

Teacher Preparation

- Read and review the background information about the bases *gon-* and *angl-, angul-* on page 61 to familiarize yourself with words associated with "angle" and "corner."
- Read and review the glossary of the bases found on the Digital Resource CD (filename: rootsglossary.pdf) for additional support.

Guided Practice

About the Root

1. Distribute copies of pages 62, 63, and/or 64 to students.
2. Write *gon-* and *angl-, angul-* = "angle," "corner" on the board. Ask students to complete the first section on page 62. After a few minutes, invite sharing.
3. Ask pairs of students to read the rest of the passages and answer the questions. You may want to conclude with whole-class discussion. Students could brainstorm words containing the roots, for example, and you could post these in the classroom.

Divide and Conquer

4. Distribute copies of *Divide and Conquer: Words Associated with Angle and Corner* (page 65). As you guide students through the activity page, use questions like these to generate discussion about each of the words:
   - Where is the meaning of "angle" or "corner" in the word _____?
   - Where might you see the word _____?
   - Can you think of an example of _____?

Bases *gon-* and *angl-, angul-* *(cont.)*

Making Connections

5. Distribute copies of *Making Connections: Riddles* (page 66).

6. You may wish to write a riddle together with students. You might conclude with a discussion that focuses on root meaning.

Words with *gon-* and *angl-, angul-*

anchor
angle
angular
ankle
decagon
diagonal
dodecagon
equiangular
heptagon
hexagon
hexagonal
nonagon
octagon
octagonal
pentagon
polygon
polygonal
quadrangle
quadrangular
rectangle
rectangular
triangle
triangular
trigonometric
trigonometry

A list of words to print out for students can be found on the Digital Resource CD (filename: wordlists.pdf).

Teacher Background Information and Tips

Bases *gon-* and *angl-, angul-* = "angle," "corner"

The Greek base *gon-* and Latin base *angl-, angul-,* meaning "angle," "corner," appear in many words from geometry. In the original ancient languages, even before the development of geometry, these bases meant "bend," generating such non-math words as *ankle* (the joint at which the leg and foot meet, creating an *angular* bend) and *anchor* (originally, a bent piece of hooked metal dropped to the bottom of the sea to dig into the sand and stabilize the boat). The original meaning of "bend," "bent" can be helpful to students in visualizing an *angle:* an *angle* is created by two lines that intersect and form a corner at the *vertex* (see Unit I, Lesson 5), the "turning point" at which the line "bends."

The concept of an *angle* as a bent line can be illustrated by drawing a *right angle*. Without lifting pencil from paper (or marker from board), draw a vertical line (from top down) and then bend it at 90 degrees into a horizontal line. By drawing students' attention to the "bent corner," you can focus their attention on the *angle* itself. When we name *angles*, such as $\angle$ ABC, the center letter always identifies the *vertex*, or "turning point" of the two rays that bend at this spot.

When studying *angles*, students often learn the *classifications of angles*. Angles are classified in terms of 90 degrees, 180 degrees, and their variance from these measurements. Because a *polygon*, by definition, contains many sides and angles, students studying *angles* and *polygons* often learn the terminology of angles as they relate to one another.

This mnemonic may help students distinguish *complementary* from *supplementary* angles. Both *supplementary* and *straight* begin with the letter *s*. *Supplementary* angles combine to form a *straight* angle (the 180 degrees that constitute a *straight* line). Both *corner* and *complementary* begin with the letter *c*. *Complementary* angles combine to form the *corner* of a right angle.

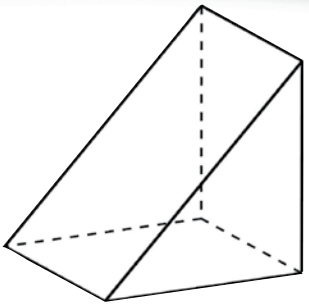
For a glossary of words built on these roots, see the Digital Resource CD (filename: rootsglossary.pdf).

Name: _____ Date: _____

About the Root:
Angles and Corners

Directions: Read the information below. Then answer the questions.

The bases *gon-* and *angl-, angul-* mean "angle," "corner." These bases appear in many words from geometry. You may already know, for example, such words as *polygon* and *hexagon.* Because a *hexagon* contains six angles, it also contains six sides. In general, a *polygon* has as many sides as it has *angles*. A geometric figure that consists of straight sides and *angles* always has an *angular* shape.



- *Poly-* means "many." What is a *polygon*? Give an example of a polygon, and then write how you know it is one.

_____

_____

_____

- *Octa(o)-* means "eight." How many corners does an *octagonal* figure have?

_____

- On many college campuses, buildings are arranged around an open, grassy area called a *quadrangle*. How many corners and sides would you expect to find in a *quadrangle* flanked by classroom buildings?

_____

_____

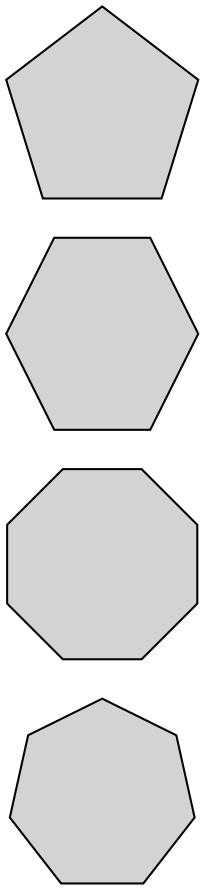
Name: _____ Date: _____

About the Root:
Polygons

Directions: Read the passage below. Then answer the questions.

Polygons have many corners. They are flat, two-dimensional figures. They have been studied since ancient times. The word *polygon* is a generic term. It refers to any two-dimensional figure that has many sides or corners. A root that means a number is added to *-gon* to name a specific polygon. For example:

- *Penta-* means "five," so a *pentagon* has five corners.
- *Hexa-* means "six," so a *hexagon* has six corners.
- *Hepta-* means "seven," so a *heptagon* has seven corners.
- *Octa-* means "eight," so an *octagon* has eight corners.



Regardless of the number of corners, all polygons share some characteristics. They are all formed from straight lines called *segments* or *edges*. The segments are linked together to form a closed chain. A vertex (plural: vertices) is the point where two segments meet. The line segments or rays that extend from vertices form angles; all polygons have angles.

Polygons also have differences aside from their numbers of corners. For example, some polygons are simple, meaning that their segments do not intersect, except at their corners. Others, like stars, are complex with segments that do intersect. Some polygons are cyclic. This means that their corners could be located on a circle.

- Is a square a simple polygon or a complex polygon? Why?

_____

_____

- Is a square a cyclic polygon? Why?

_____

_____

Name: _____ Date: _____

About the Root:
Triangles

Directions: Read the passage below. Then answer the questions on a separate sheet of paper.

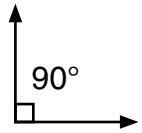
A *triangle* is a two-dimensional polygon. Along with circles and rectangles, triangles are considered basic shapes. Triangles have three sides, sometimes called edges or segments. The sides are closed. They also have three angles or corners, also called *vertices*. Euclid described the basic facts and properties of triangles in 300 B.C.

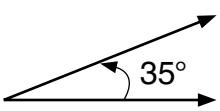
The names of triangles depend on their shapes. Either the length of the sides or the size of the inside angles determines the type of triangle. For example, an equilateral triangle has three equal sides. *Equi-* means "equal" and *later-* means "side," so this is an easy one to remember. An *isosceles* triangle has at least two equal sides. If you know about *isobars* on weather maps, which connect places with equal atmospheric pressure, this may help you remember *isosceles*. (*Iso-* means "equal," and Greek *scel-* means "leg," "ankle," referring to the leglike sides of a triangle.) *Scalene* triangles have uneven sides.

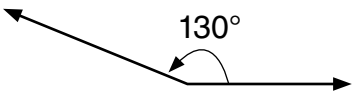
Triangles can also be labeled according to their angles. The inside angles of a triangle always equal 180 degrees. An *equiangular* triangle has, as you might guess, equal angles. Each angle is 60 degrees. A *right triangle* has one 90 degree angle. *Acute* triangles have angles that are less than 90 degrees. *Obtuse* triangles have an angle greater than 90 degrees. Remembering that *A* comes before *O* in the alphabet may help you remember that acute triangles have smaller angles than obtuse triangles.

◎ Is an equilateral triangle also an isosceles triangle? How do you know?

◎ Could an obtuse triangle also be a right triangle? How do you know?







Name: _____ Date: _____

Divide and Conquer:

Words Associated with Angle and Corner

Directions: Use the Base/Prefix/Suffix Bank to help you identify the roots of the words below. Write the letter from the Definition Bank for the correct word. The *-al* suffix makes three of the words adjectives.

Base/Prefix/Suffix Bank

dia- = "through," "across"
hex(a)- = "six"
oct(a)- = "eight"
quadr- = "four"
rect- = "right"

Definition Bank

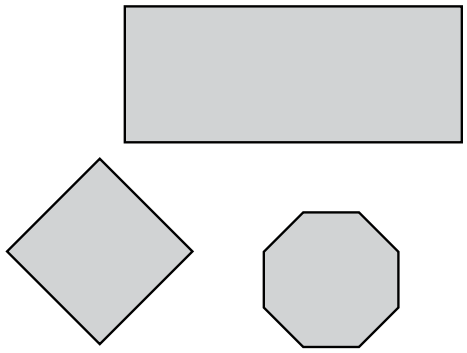
A. a polygon with four angles of 90 degrees each

B. having six sides or angles

C. any four-sided polygon

D. having eight sides or angles

E. a line cutting across a polygon and connecting two nonadjacent angles



Word	First Base/Prefix Means	Second Base/Suffix Means	Definition
1. quadrangle			
2. diagonal			
3. rectangle			
4. octagonal			
5. hexagonal			

Name: _____ Date: _____

Making Connections:
Riddles

Directions: Make riddles for two of the terms in the Word Bank. Leave the last line blank. Then give your riddles to a classmate to solve. If you would like to create more riddles, use a separate sheet of paper.

Example:

I am a figure with many angles.

I have five letters; three are vowels.

The top of a child's wagon is my shape.

The top of a pizza box is my shape.

What am I?

Answer: a square

Word Bank

polygon	complex polygon	triangle
simple polygon	rectangle	term

1. _____

_____

_____

_____

_____

_____

_____

2. _____

_____

_____

_____

_____

_____

_____

