Up and Down Stretch

Standard

- Adds, subtracts, multiplies, and divides integers, and rational numbers

Overview

Students investigate the rules for multiplying integers.

Materials

- chart paper
- markers
- scientific or graphing calculator

Warming Up for the Stretch

Prior to doing this Stretch, students should have learned about integers and how they are ordered. They should have performed operations with negative numbers with and without calculators.

Up and Down Stretch *(cont.)*

Stretch Procedure

1. Prepare a class chart with the problems included for investigation of multiplying positive and negative integers. (See sample on page 43.)
2. Display the chart as students enter the room.
3. Have the students choose a problem and write the answer in the appropriate place on the chart paper.
4. When all students have finished entering their answers on the chart, meet as a class in a Math Huddle. You can use the questions below to guide this discussion.

Suggested Questions for Informal Assessment: Math Huddle

Level of Teacher Support

- What do you notice about how the products change in problems 1–9? (Students should notice as the product decreases by 4, the second factor decreases by 1.) Why does this occur? What does this tell you about multiplying a positive number by a negative number?
- How do the products change in problems 10–18? Why do you think the answers to 10–14 are the same, but in reverse order, as the answers to 6–9? Read further down the chart; what is happening to the value of each product? What does this tell you about multiplying a negative number by a negative number?
- How are problems 19–27 different from the others? Which problems have positive answers? How many negative numbers were used as factors in each of these problems? Was there an even or odd number of negative factors?
- Which problems have negative answers? How many negative numbers were used as factors in each of these problems? Was there an even or odd number of negative factors?
- What is the rule for multiplying several negative numbers? Do these same rules work if there are just two factors?
- Do you think this rule would work if the problem had a mixture of positive and negative numbers? Why do you think it would or would not work? Think of a problem to check on your calculator.

Up and Down Stretch *(cont.)*

What It Looks Like: Stretch Snapshot

This Stretch Snapshot helps students discover the patterns that explain the basic rules for determining the sign of the product when multiplying two integers. Generating the rules for multiplying two integers should be easy and will help them to recognize that, given more than two factors, the product is negative if there are an odd number of negative integers, but positive if there are an even number of negative integers. Using the associative property to group numbers is sometimes useful. For example, 2 x –3 x –4 x 5 x –6 can be multiplied as 2 x –3 = –6, then –6 x –4 = 24, then 24 x 5 = 120, and 120 x –6 = –720. This shows that multiplication is a binary operation, and each negative sign determines the sign of one of the multiplication problems.

In this Math Huddle, the teacher feels that Elena does not completely understand how to determine the sign of a multiplication problem that contains more than two factors.

Teacher: *Elena, think about the products we found for the problems on the chart. From the patterns we discovered, how could you determine whether the answer to a multiplication problem is positive or negative?*

Elena: *If there is one negative, the answer is negative. If there are two negatives, the answer is positive.*

Teacher: *Does everyone agree?*

Trent: *I do. Look at the chart. Every time we have just one negative factor, the product is negative. When there are two negative factors, the products are always positive-except for problems like Elena's problem, 2 x –3 x –4.*

Teacher: *Elena, can you find two problems where Trent's rule works?*

Elena: *Yes, 2 x –3 = –6 because there is one negative. And –2 x –3 = 6 because there are two negatives.*

Teacher: *Does everyone agree with Elena? (Students nod.) I think you understand how to multiply two numbers together. What do you do if there are more than two factors in the multiplication problem? Turn to your elbow partner and share what you think.* (The teacher gives students time to share their ideas.) *Now, Elena, what do you think?*

Elena: *You count and see if it's even or odd.*

Elena: *You count the numbers and see if it's even or odd.*

Teacher: *Are you saying that we count how many numbers are being multiplied, the total number of factors? Let's look at your problem on the chart: 2 x –3 x –4.*

Up and Down Stretch *(cont.)*

What It Looks Like: Stretch Snapshot *(cont.)*

Elena: *I think that 2 x –3 x –4 should be –24 because there are three numbers.*

Trent: *I don't think it will be negative.*

Teacher: *Why not?*

Trent: *Well, I remember that you can arrange the numbers in a multiplication problem in any order that you want. If you multiplied 2 x –3 first, you get –6 because there is one negative.*

Teacher: *Okay. Where do you go from there?*

Trent: *Now I would multiply that answer, –6, times –4.*

Elena: *Hey, that's 24, because there are two negatives.*

Teacher: *Elena, what do you think? Look at the original problem. How many negatives are there?*

Elena: *Two. Oh, it's the number of negatives we multiply—not all the numbers that are multiplied!*

Teacher: *Let's try one. Think about 2 x –3 x 4 x 2 x –3 x –4. Multiply the numbers out two at a time and then multiply your answers together.*

Elena: *2 x –3 is –6; 4 x 2 is ;, and –3 x –4 is 12; –6 x 8 is –48; and –48 x 12 is –576.*

Teacher: *Why do you think the product is negative?*

Elena: *There are three negatives. Three is an odd number.*

Teacher: *What if you changed one of the positive numbers to a negative number? How many negatives would there be?*

Elena: *There would be four negatives.*

Teacher: *Would the answer be positive or negative?*

Elena: *Positive.*

Teacher: *So, mathematicians, anytime that you are multiplying a number of factors—some of which are positive and some of which are negative—what did we discover? How can we know if the product will be positive or negative?*

Elena: *Count the negatives. An odd number of negatives means that answer is negative. An even number of negatives means that the answer is positive.*

Teacher: *Good job, Elena. That's exactly right. Always remember to use that rule when you are multiply positive and negative factors.*

Up and Down Stretch *(cont.)*

Sample Chart

Choose a problem below and solve it with and without a calculator. If the answers are different, put both answers on the chart. Add your initials.

1. 4 x 4 = 16 BG	10. –4 x 4 = -16 NB
2. 4 x 3 = 12 KM	11. –4 x 3 = -12 YM
3. 4 x 2 = 8 LI	12. –4 x 2 = -8 JB
4. 4 x 1 = 4 CY	13. –4 x 1 = -4 JL
5. 4 x 0 = 0 MK	14. –4 x 0 = 0 CJ
6. 4 x –1 = -4 ED	15. –4 x –1 = 4 RT
7. 4 x –2 = -8 SP	16. –4 x –2 = 8 MT
8. 4 x –3 = -12 QW	17. –4 x –3 = 12 DM
9. 4 x –4 = -16 ZS	18. –4 x –4 = 16 DB

19. 2 x 3 x 4 has __0__ negative signs and the answer is __24 GG__.

20. 2 x 3 x –4 has __1__ negative signs and the answer is __-24 NY__.

21. 2 x –3 x –4 has __2__ negative signs and the answer is __-24 ER__.

22. –2 x –3 x –4 has __3__ negative signs and the answer is __-24 GG__.

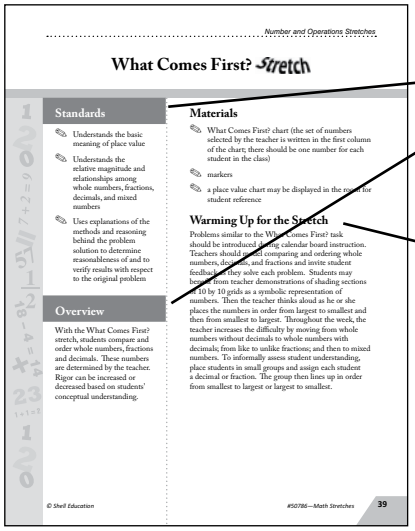
23. –1 x –2 x –3 x –4 has __4__ negative signs and the answer is __24 BM__.

24. –1 x –2 x –3 x –4 x –5 has __5__ negative signs and the answer is __-120 SY__.

25. –1 x –2 x –3 x –4 x –5 x –6 has __6__ negative signs and the answer is __750 SE__.

26. –1 x –2 x –3 x –4 x –5 x –6 x –7 has __7__ negative signs and the answer is __-5040 GG__.

How to Use This Product

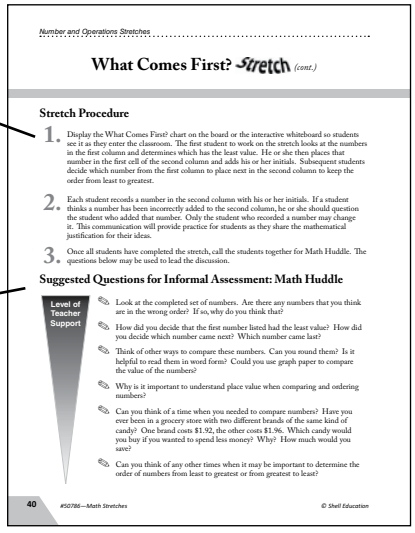


Each section opens with a list of the standards that are represented by the activity, followed by an overview of the stretch.

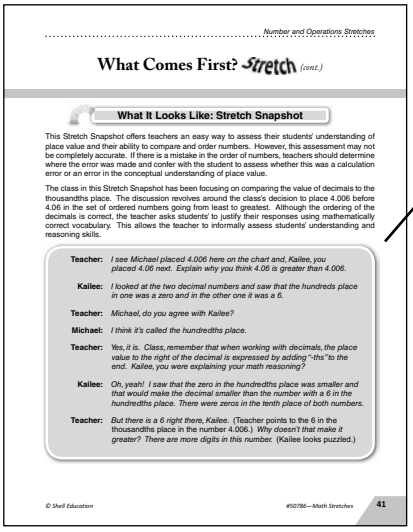
A list of any materials and necessary prerequisite instruction (Warming Up for the Stretch) are included to help the teacher prepare the classroom and the students for the activity, minimizing the need for teacher assistance and allowing the students to have as much independence as possible to complete the task.

A simple, step-by-step procedure directs the teacher in how to conduct the Math Stretch. Included in this section are suggestions for extending the stretch for further mathematical exploration, as well as modifications for students who are nonreaders.

The Math Huddle section suggests questions for informal assessment that a teacher can ask to provide varying levels of support and to facilitate a gradual release of responsibility (see more about Inquiry-based Learning on page 33).



How to Use This Product *(cont.)*



Number and Operations Stretches

What Comes First? Stretch *(cont.)*

What It Looks Like: Stretch Snapshot

This Stretch Snapshot offers teachers an easy way to assess their students' understanding of place value and their ability to compare and order numbers. However, this assessment may not be completely accurate. If there is a mistake in the order of numbers, teachers should determine where the error was made and confer with the student to assess whether this was a calculation error or an error in the conceptual understanding of place value.

The class in this Stretch Snapshot has been focusing on comparing the value of decimals to the thousandths place. The discussion revolves around the class's decision to place 4.006 before 4.06 in the set of ordered numbers going from least to greatest. Although the ordering of the decimals is correct, the teacher asks students' to justify their responses using mathematically correct vocabulary. This allows the teacher to informally assess students' understanding and reasoning skills.

Teacher: *I see Michael placed 4.006 here on the chart and, Kailee, you placed 4.06 next. Explain why you think 4.06 is greater than 4.006.*

Kailee: *I looked at the two decimal numbers and saw that the hundreds place in one was a zero and in the other one it was a 6.*

Teacher: *Michael, do you agree with Kailee?*

Michael: *I think it's called the hundredths place.*

Teacher: *Yes, it is. Class, remember that when working with decimals, the place value to the right of the decimal is expressed by adding "-ths" to the end. Kailee, you were explaining your math reasoning?*

Kailee: *Oh, yeah! I saw that the zero in the hundredths place was smaller and that would make the decimal smaller than the number with a 6 in the hundredths place. There were zeros in the tenth place of both numbers.*

Teacher: *But there is a 6 right there, Kailee.* (Teacher points to the 6 in the thousandths place in the number 4.006.) *Why doesn't that make it greater? There are more digits in this number.* (Kailee looks puzzled.)

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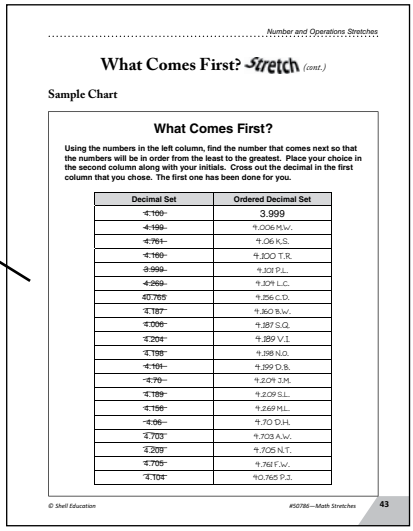
The Math Stretch concludes by providing the teacher with a model of how the Math Huddle may look in an actual classroom setting. This Stretch Snapshot illustrates the kinds of conversations teachers can have with their students, demonstrating how to extend students' thinking or uncover the sources of students' confusion about a concept. These dialogues model guided inquiry, in which the teacher facilitates the conversation, so that students can make connections and discover underlying themes on their own.

A sample of the chart or poster created during each stretch is also provided. The specific information shown in each sample mirrors the content of the Stretch Snapshot.

Templates of each chart or poster are available on the Teacher Resource CD, as well as electronic formats using interactive whiteboard technology.



Number and Operations Stretches

What Comes First? Stretch *(cont.)*

Sample Chart

What Comes First?

Using the numbers in the left column, find the number that comes next so that the numbers will be in order from the least to the greatest. Place your choice in the second column along with your initials. Cross out the decimal in the first column that you chose. The first one has been done for you.

Decimal Set	Ordered Decimal Set
~~4.100~~	3.999
~~4.199~~	4.006 M.W.
~~4.761~~	4.06 K.S.
~~4.160~~	4.100 T.R.
~~3.999~~	4.101 P.L.
~~4.269~~	4.104 L.C.
~~40.765~~	4.156 C.D.
~~4.187~~	4.160 B.W.
~~4.006~~	4.187 S.Q.
~~4.204~~	4.189 V.I.
~~4.198~~	4.198 N.O.
~~4.159~~	4.199 D.B.
~~4.270~~	4.204 J.M.
~~4.189~~	4.209 S.L.
~~4.156~~	4.269 M.L.
~~4.06~~	4.70 D.H.
~~4.703~~	4.703 A.W.
~~4.209~~	4.705 N.T.
~~4.705~~	4.761 F.W.
~~4.104~~	40.765 P.J.

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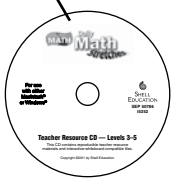

Guided Math: A Flexible Framework for Mathematics Instruction

As teachers, we struggled to find practical ways to incorporate best practices into my mathematics instruction. Gradually, I developed a model that allowed me to offer every student an opportunity to develop his or her mathematical skills at increasingly challenging levels of difficulty. My ultimate goal was to help each gain the ability to function independently in the world of mathematics. The Guided Math framework I used is designed to support mathematical literacy by mirroring many of the same techniques applied to teaching literacy for many years.

Guided Math is broadly defined as a flexible instructional framework that enables teachers to promote the deep mathematical understanding and computational fluency of their students by determining their unique needs and prescriptively addressing those needs through a combination of whole-class instruction, small-group instruction, math workshops, and conferences within a classroom environment supportive of numeracy.

The specific instructional components of this model include:

1. A Classroom Environment of Numeracy
2. Morning Math Warm-Ups and Calendar Board Activities
3. Whole-Class Instruction
4. Guided Math Instruction with Small Groups of Students
5. Math Workshops
6. Individual Conferences
7. An Ongoing System of Assessment

Used together, these components allow teachers to implement research-based best practices in their classrooms that support each student's mathematical learning according to his or her needs.

A Classroom Environment of Numeracy

Environments rich in mathematical opportunities are essential if students are to develop a thorough understanding of mathematics. When students begin to recognize how numbers and problem solving affect their everyday lives, mathematics becomes more meaningful to them. Because learning is both a social and constructive process, children learn best through active engagement in authentic tasks that offer opportunities to use and extend their number senses.

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