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Prime Factorization

A number that has only two factors, 1 and itself, is called a *prime number*. Numbers such as 2, 3, 7 and 11 are prime numbers. A number that has more than two factors is a *composite number*. Numbers such as 4, 8, 9, and 15 are composite numbers.

You can write any composite number as a product of prime numbers. For example, you can write 18 as the product of several prime numbers.

$$18 = 2 \times 9$$

prime number
composite number

As you can see 9 is also a composite number. You can factor 9 to 3×3 .

$$18 = 2 \times 9 = 2 \times 3 \times 3$$

Rules for Prime Factorization

1. Find two factors of the number.
2. Determine if the factors are prime.
3. Factor the composite numbers again.
Repeat until you have only prime numbers.

Example

Find the prime factorization of 20.

- Step 1** Find two factors of the number. $20 = 5 \times 4$
- Step 2** Determine if the factors are prime or composite numbers.
 5 is a prime number;
 4 is a composite number.
- Step 3** Factor the composite numbers again. $4 = 2 \times 2$, so $20 = 5 \times 4 = 5 \times 2 \times 2$
 All the factors are now prime numbers.

Practice

Find the prime factorization of each number.

1. 32

- Find two factors of the number. $32 = 2 \times \underline{\hspace{2cm}}$
- Determine if the factors are prime or composite numbers.
 2 is _____; _____ is composite.
- Factor the composite numbers again. $32 = 2 \underline{\hspace{1cm}} \underline{\hspace{1cm}}$
- Repeat until you have only prime numbers.
 $32 = 2 \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}}$
 $32 = 2 \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}}$
 $32 = 2 \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}} \underline{\hspace{1cm}}$

2. $30 = \underline{\hspace{2cm}}$ 5. $18 = \underline{\hspace{2cm}}$
3. $24 = \underline{\hspace{2cm}}$ 6. $81 = \underline{\hspace{2cm}}$
4. $15 = \underline{\hspace{2cm}}$ 7. $100 = \underline{\hspace{2cm}}$

Least Common Multiple

A *multiple* of a number is the product of the number and a whole number. For example, multiples of 4 are:

$4 \times 0 = 0$

$4 \times 1 = 4$

$4 \times 2 = 8$

$4 \times 3 = 12$

$4 \times 4 = 16$

$4 \times 5 = 20$

Therefore the multiples of 4 are 0, 4, 8, 12, 16, 20, and so on. A *common multiple* of two different numbers is a number that is a multiple of both of those numbers. For example, 12 is a multiple of 3, 4, and 6.

The *least common multiple* (LCM) is the smallest common multiple of two numbers (not including 0).

Rules for Finding the Least Common Multiple

1. List all the multiples of each number.
2. Find the smallest number (other than zero) that is the same in each list. That is your least common multiple.

Example

Find the least common multiple of 3 and 5.

Step 1 List all of the multiples of each number.

Multiples of 3: 0, 3, 6, 9, 15, 18

Multiples of 5: 0, 5, 10, 15, 20

Step 2 Find the smallest number (other than zero) that is the same on each list.

The smallest multiple common to 3 and 5 is 15.

Practice

1. Find the least common multiple of 4 and 6.

List all of the multiples of each number.

Multiples of 4: 0, 4, 8, 12, 16, 20

Multiples of 6: _____

Find the smallest number (other than zero) that is the same on each list.

The smallest number on each list is _____,
so the LCM is _____.

List the first six multiples of each number.

2. 3 _____

3. 7 _____

4. 10 _____

Find the least common multiple of each pair of numbers.

5. 2 and 6 _____

8. 6 and 9 _____

6. 4 and 5 _____

9. 9 and 12 _____

7. 3 and 7 _____

10. 8 and 10 _____

Greatest Common Factor

The numbers that you multiply are called *factors*. The result, or the answer of a multiplication sentence, is called the *product*. There can be several factors that you can multiply to get a certain number. For example, the factors of 12 are found by thinking of all the combinations of two numbers that when multiplied will equal 12.

$$1 \times 12 = 12 \quad 2 \times 6 = 12 \quad 3 \times 4 = 12$$

The factors of 12: 1, 2, 3, 4, 6

A number can be a factor in two different numbers. For example, 3 is a factor of 9 ($3 \times 3 = 9$) and 15 ($3 \times 5 = 15$).

The largest common factor of two numbers is called the *greatest common factor* (GFC).

Rules for Finding the Greatest Common Factor

1. List the multiples (factors) of each number.
2. Find the numbers that are the same on both lists.
3. Of the numbers that are the same, find the largest number.

Example

Find the greatest common factor of 12 and 18.

Step 1 List all the multiples (factors) of each number.

Multiples of 12: 1, 2, 3, 4, 6, 12

Multiples of 18: 1, 2, 3, 6, 9, 18

Step 2 Find the numbers that are the same in each list.

The numbers that are the same are 1, 2, 3, 6.

Step 3 Of the numbers that are the same, find the largest number.

The largest number is 6, so 6 is the greatest common factor of 12 and 18.

Practice

1. Find the greatest common factor of 8 and 14.

List all the multiples (factors) of each number.

Multiples of 8: _____

Multiples of 14: _____

Find the numbers that are the same in each list.

The numbers that are the same are _____ and _____.

Of the numbers that are the same, find the largest number.

The largest number is _____, so _____ is the greatest common factor of 8 and 14.

List the factors of each of the numbers.

Find the greatest common factor (GCF).

2. 10 _____

5. 16 and 24 _____

3. 24 _____

6. 10 and 18 _____

4. 30 _____

7. 22 and 44 _____

Exponents

You can show the repeated multiplication of the same number using *exponents*.
In an expression such as 4^3 , the “4” is known as the *base*, and the “3” is the *exponent*.

Rules for Working with Exponents

To solve an expression with an exponent:

Multiply the base by itself the number of times equal to the exponent.

To write an expression using an exponent:

Count the number of times a number is multiplied by itself;
that amount is your exponent.

The number being multiplied is the base.

Example

Solve the following expression. 5^3

Multiply the base by itself the number of times equal to the exponent.

The exponent is 3, so you multiply 5 by itself 3 times: $5^3 = 5 \times 5 \times 5 = 125$.

Write $6 \times 6 \times 6 \times 6$ using an exponent.

Step 1 Count the number of times a number is multiplied by itself, that amount is your exponent.

6 is multiplied by itself 4 times; the exponent is 4.

Step 2 The number being multiplied is the base.

6 is being multiplied by itself, so 6 is the base: $6 \times 6 \times 6 \times 6 = 6^4$.

Practice

1. Solve the following expression. 2^6

Multiply the base by itself the number of times equal to the exponent.

$$2 \times 2 \times 2 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

2. Write the expression $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ using an exponent.

Count the number of times a number is multiplied by itself; that amount is your exponent.

_____ is multiplied by itself _____ times.

The number being multiplied is the base.

$$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = \underline{\hspace{2cm}}$$

Solve the following expressions.

3. $3^5 = \underline{\hspace{2cm}}$

4. $12^2 = \underline{\hspace{2cm}}$

5. $8^3 = \underline{\hspace{2cm}}$

Write the following expressions using an exponent.

6. $10 \times 10 \times 10 \times 10 = \underline{\hspace{2cm}}$

7. $6 \times 6 \times 6 = \underline{\hspace{2cm}}$

8. $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = \underline{\hspace{2cm}}$

Exponents and Multiplication

When multiplying two expressions with exponents where the base is the same, you follow a couple of simple rules. Look at the examples below.

$$6^3 \times 6^5 = 6^8 \quad 12^2 \times 12^{12} = 12^{14} \quad 7^2 \times 7^8 = 7^{10}$$

Rules for Exponents and Multiplication

1. Add the exponents. The sum is your new exponent.
2. Keep the base the same.

Example

Multiply. $5^2 \times 5^3$

Step 1 Add the exponents. The sum is the new exponent.

$$2 + 3 = 5$$

Step 2 Keep the base the same.

$$5^2 \times 5^3 = 5^{2+3} = 5^5$$

Practice

Multiply.

1. $10^3 \times 10^5$

Add the exponents. The sum is the new exponent.

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

Keep the base the same.

$$10^3 \times 10^5 = 10^{\underline{\quad}} = 10^{\underline{\quad}}$$

2. $2^4 \times 2^3 =$ _____

3. $5^1 \times 5^0 =$ _____

4. $6^4 \times 6^{10} =$ _____

5. $12^2 \times 12^{15} =$ _____

6. $8^4 \times 8^4 =$ _____

7. $9^3 \times 9^6 =$ _____

8. $10^{10} \times 10^5 =$ _____

9. $5^4 \times 5^{-2} =$ _____

10. $12^{10} \times 12^{-5} =$ _____

11. $6^7 \times 6^{-7} =$ _____

Exponents and Division

When dividing two expressions with exponents where the base is the same, you follow a couple of simple rules. Look at the examples below.

$$6^5 \div 6^2 = 6^3 \quad 12^9 \div 12^3 = 12^6 \quad \frac{9^{16}}{9^9} = 9^7$$

Rules for Exponents and Division

1. Subtract the exponent in the divisor from the exponent in the dividend.
2. Keep the base the same.

Example

Divide. $5^6 \div 5^2$

Step 1 Subtract the exponents. The sum is the new exponent. $6 - 2 = 4$

Step 2 Keep the base the same. $5^6 \div 5^2 = 5^{6-2} = 5^4$

Practice

Divide.

1. $6^7 \div 6^5$

Subtract the exponents. The sum is the new exponent. _____ - _____ = _____

Keep the base the same. $6^7 \div 6^5 = 6^{\text{—————}} = 6^{\text{—————}}$

2. $2^8 \div 2^2 =$ _____

3. $16^5 \div 16^1 =$ _____

4. $8^8 \div 8^0 =$ _____

5. $9^7 \div 9^7 =$ _____

6. $12^{10} \div 12^4 =$ _____

7. $7^{10} \div 7^{-4} =$ _____

8. $4^5 \div 4^{-2} =$ _____

9. $6^{-2} \div 6^2 =$ _____

10. $\frac{13^4}{13^2} =$ _____

11. $\frac{8^{12}}{8^{-10}} =$ _____

Scientific Notation

A shorthand way to write a large number or small number is to use *scientific notation*.

$$3,400 \rightarrow 3.4 \times 10^3 \quad 0.00923 \rightarrow 9.23 \times 10^{-3}$$

As you can see, a number in scientific notation is made of a number between 1 and 10 multiplied by 10 raised to a power.

Rules for Using Scientific Notation

1. Move the decimal point to the left or right to get a number between 1 and 10.
2. Multiply that number by 10 with an exponent.
3. The exponent is equal to the number of places the decimal point moved.
4. The exponent is positive if the decimal point is moved to the left; negative if moved to the right.

Example

Write 462,000 in scientific notation.

- | | | |
|---------------|---|--|
| Step 1 | Move the decimal point to the left or right to get a number between 1 and 10. | 462,000 (5 decimal places): 4.62 |
| Step 2 | Multiply the number by 10 with an exponent. | $4.62 \times 10^?$ |
| Step 3 | The exponent is equal to the number of places the decimal point moved. | The decimal point moved 5 places.
4.62×10^5 |
| Step 4 | The exponent is positive if the decimal point is moved to the left; negative if moved to the right. | The decimal point moved to the left.
4.62×10^5 |

Practice

Write each number in scientific notation.

1. 0.000433

Move the decimal point to the left or right to get a number between 1 and 10. 0.000433 (_____ decimal places): 4.33

Multiply the number by 10 with an exponent. 4.33 _____

The exponent is equal to the number of places the decimal point moved. 4.33×10 _____

The exponent is positive if the decimal point is moved to the left; negative if moved to the right. 4.33×10 _____

2. 25,000 _____

5. 0.015 _____

3. 4,000,000 _____

6. 0.000791 _____

4. 663,200 _____

7. 0.0000042 _____

Square Roots

When you multiply a number by itself (for example, 4×4), you *square* the number (in this case, 4). The opposite of squaring a number is to find the *square root* of a number. The square root of a given number is the number that, when squared, results in the given number.

For example, 4 squared is 16 ($4 \times 4 = 16$). The square root of 16 is 4 ($\sqrt{16} = 4$). As you can see, the square root of a number uses the square root symbol ($\sqrt{\quad}$) and the number.

Rules for Finding the Square Root

1. Look at the number under the square root symbol. Use guess and test, or a table of squares or square roots to find the square root. **Or**
2. Using a calculator, enter a number, press the square root key ($\sqrt{\quad}$), and equals (=) sign.
3. The square root of any positive number can be either positive or negative; you must include both possibilities in your answer.

Example

What is $\sqrt{144}$?

Step 1 Look at the number under the square root symbol. You want to find the square root of 144.

Step 2 Use guess and test, or a square root table. You know that $10 \times 10 = 100$, so 144 will be greater than 10. By guess and test, you find $12 \times 12 = 144$. So, $\sqrt{144} = 12$.

Step 3 The square root of any positive number can be either positive or negative. The square root is either 12 or -12 .

Practice

Find the square root of the following.

1. $\sqrt{64}$

Look at the number under the square root symbol.

You need to find the square root of _____ or find what number _____ by itself equals _____.

Use guess and test, or a square root table.

You know that _____ = 25. So _____ is greater than _____. By guess and test, _____ = 64. So, $\sqrt{64} =$ _____.

The square root of any positive number can be either positive or negative.

The square root is either _____.

2. $\sqrt{121} =$ _____

4. $\sqrt{841} =$ _____

3. $\sqrt{49} =$ _____

5. $\sqrt{289} =$ _____

Cube Roots

When you multiply a number by itself three times (for example, $4 \times 4 \times 4$), you *cube* the number (in this case, 4). The opposite of cubing a number is to find the *cube root* of a number. The cube root of a given number is the number that, when cubed, results in the given number.

For example, 4 cubed (4^3) is 64 ($4 \times 4 \times 4 = 64$). The cube root of 64 is 4 ($\sqrt[3]{64} = 4$). As you can see, the cube root of a number uses the cube root symbol ($\sqrt[3]{\quad}$) and the number.

Rules for Finding the Cube Root

1. Look at the number under the cube root symbol. Use guess and test, or a table of squares or cube roots to find the cube root. **Or**
2. Using a calculator, enter a number, press the ($\sqrt[3]{y}$) key, “3” to indicate the cube root, and equals sign (=).
3. The cube root of a positive number is positive; the cube root of a negative number is negative.

Example

What is $\sqrt[3]{27}$?

- | | |
|---|---|
| Step 1 Look at the number under the cube root symbol. | You want to find the cube root of 27. |
| Step 2 Use guess and test, or a cube root table. | You know that $2 \times 2 \times 2 = 8$, so $\sqrt[3]{27}$ will be greater than 2. By guess and test, try 3. $3 \times 3 \times 3 = 27$. So, $\sqrt[3]{27} = 3$. |
| Step 3 The cube root of any positive number is positive. | The cube root is 3. |

Practice

Find the cube root of the following.

1. $\sqrt[3]{-125}$.

Look at the number under the cube root symbol.

Use guess and test, or a cube root table.

The cube root of any negative number is negative.

You need to find the cube root of _____ or find what number _____ by itself three times is _____.

You know that _____ = 64.

So $\sqrt[3]{-125}$ is greater than _____. By guess and test _____ = 125. So, $\sqrt[3]{-125} = \underline{\hspace{2cm}}$.

The cube root is _____.

2. $\sqrt[3]{216} = \underline{\hspace{2cm}}$

4. $\sqrt[3]{-1,000} = \underline{\hspace{2cm}}$

3. $\sqrt[3]{-1} = \underline{\hspace{2cm}}$

5. $\sqrt[3]{343} = \underline{\hspace{2cm}}$

Order of Operations

Suppose you were given the following expression: $3 \times 2 + 4 \times 5 = ?$

Is the answer 50 or is it 26? To solve an expression with several operations, you need to perform your calculations in a certain order.

This order of operations lists the sequence of operations in an expression.

1. Parentheses: simplify any operations in parentheses.
2. Exponents: simplify any terms with exponents.
3. Multiply and Divide: do all multiplication and division from left to right.
4. Addition and Subtraction: do all addition and subtraction from left to right.

Example

Simplify. $4 + (4 \times 3) \div 2 \times 2^2$

Step 1 Parentheses

$$4 + (4 \times 3) \div 2 \times 2^2$$

$$4 + (12) \div 2 \times 2^2$$

Step 2 Exponents

$$4 + (12) \div 2 \times 2^2$$

$$4 + (12) \div 2 \times 4$$

Step 3 Multiplication and division

$$4 + (12) \div 2 \times 4$$

$$4 + 6 \times 4$$

$$4 + 24$$

Step 4 Addition and subtraction

$$4 + 24 = 28$$

$$4 + (4 \times 3) \div 2 \times 2^2 = 28$$

Practice

Simplify each expression.

1. $(5 \times 7) + 3^2 - 8 \div 4$

Parentheses

$$(5 \times 7) + 3^2 - 8 \div 4$$

$$\underline{\hspace{2cm}} + 3^2 - 8 \div 4$$

Exponents

$$\underline{\hspace{2cm}} + 3^2 - 8 \div 4$$

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} - 8 \div 4$$

Multiplication and division

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} - 8 \div 4$$

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$$

Addition and subtraction

$$\underline{\hspace{2cm}} - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$(5 \times 7) + 3^2 - 8 \div 4 = \underline{\hspace{2cm}}$$

2. $1 + 8 \div 2^2 \times 4 = \underline{\hspace{2cm}}$

5. $(10 - 3) \times 2^2 + 4 = \underline{\hspace{2cm}}$

3. $5 + 6 \times 3 \div 3^2 - 2 = \underline{\hspace{2cm}}$

6. $24 \div (6 - 3) + 4^2 = \underline{\hspace{2cm}}$

4. $(2^3 - 3) \times 5 - 1 = \underline{\hspace{2cm}}$

7. $(5 + 8 \times 2) + 3 \times 2 = \underline{\hspace{2cm}}$

Distributive Property

Suppose you have the expression $2 \times (4 + 5)$ or $2(4 + 5)$. In this expression you are using two operations, multiplication and addition. You can rewrite the expression using the *Distributive Property*. When you use the Distributive Property, you distribute the number outside the parentheses to each number inside the parentheses.

Rules for the Distributive Property

1. Multiply the number outside the parentheses by each number inside the parentheses.
2. Place the operation symbol inside the parentheses between the two multiplication expressions.
3. Simplify using order of operations.

Example

Simplify using the Distributive Property. $2(6 + 3)$

Step 1 Multiply the number outside the parentheses by each number inside the parentheses. $2(6 + 3) = 2 \times 6$ and 2×3

Step 2 Place the operation symbol inside the parentheses between the two multiplication expressions. 2×6 and $2 \times 3 = 2 \times 6 + 2 \times 3$

Step 3 Simplify using the order of operations. $2 \times 6 + 2 \times 3 = 12 + 6 = 18$

Practice

Use the Distributive Property to simplify each expression.

1. $5(3 + 2)$.

Multiply the number outside the parentheses by each number inside the parentheses.

$$5(3 + 2) = 5 \times 3 \underline{\hspace{2cm}}$$

Place the operation symbol inside the parentheses between the two multiplication expressions.

$$5 \times 3 \underline{\hspace{1cm}} = 5 \times 3 + \underline{\hspace{2cm}}$$

Simplify using the order of operations.

$$5 \times 3 + \underline{\hspace{1cm}} = 15 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

2. $4(3 + 2) = \underline{\hspace{2cm}}$

3. $5(6 + 7) = \underline{\hspace{2cm}}$

4. $8(2 + 4) = \underline{\hspace{2cm}}$

5. $5(4 - 2) = \underline{\hspace{2cm}}$

6. $7(12 - 6) = \underline{\hspace{2cm}}$

7. $10(9 - 3) = \underline{\hspace{2cm}}$

Divisibility Rules

If one number divides evenly into another number, then the second number is *divisible* by the first. For example, 534 is divisible by 3 because $534 \div 3 = 178$. As you can see, when one number is divided by another, there is no remainder.

Divisibility Rules: A number is divisible by

- 2 if the number is even.
- 3 if the sum of the digits is divisible by 3.
- 4 if the number formed by the last 2 digits is divisible by 4.
- 5 if the last digit is 0 or 5.
- 6 if the number is even and divisible by 3.
- 8 if the number is divisible by 4.
- 9 if the sum of the digits is divisible by 9.
- 10 if the last digit is 0.

Example

Is 198 divisible by 9?

Step 1 What is the divisibility rule?

The sum of the digits is divisible by 9.

Step 2 Apply the rule.

The sum of the digits is $1 + 9 + 8 = 18$.

Is the sum divisible by 9? Yes, $18 \div 9 = 2$.

Practice

Use the divisibility rules for the following situations.

1. Is 39 divisible by 3? by 6?

Divisible by 3

The sum of the digits is divisible by _____.

$3 + 9 =$ _____; _____ is divisible by 3.

Divisible by 6

The number is _____ and divisible

by _____. 39 _____ even.

So 39 _____ divisible by 6.

2. Is 2,160 divisible by 2? by 5? _____

3. Is 4,485 divisible by 4? by 9? _____

4. Is 9,756 divisible by 3? by 4? _____

5. Is 2,401 divisible by 3? by 9? _____

6. Is 1,234 divisible by 6? by 8? _____

7. Is 192 divisible by 4? by 6? _____

8. Is 3,725 divisible by 9? by 10? _____

9. Is 6,859 divisible by 3? by 9? _____

Number Patterns—Arithmetic Sequences

A useful skill in mathematics is to find a pattern in a series of numbers. When you look for a number pattern you check to see how the numbers change from one to the next. For example, look at the following number sequence: 5, 8, 11, 14, 17, 20.

The pattern is to add 3 to each number to get to the next. A number sequence in which a certain amount is added to or subtracted from a number to get to the next number is an *arithmetic sequence*.

Determining the rule in an Arithmetic Sequence

1. Determine how the first number is changed to get to the second number.
2. Determine how the second number is changed to get to the third number.
3. Check the next two numbers to see if the pattern continues.
4. Use the pattern to finish the sequence, if needed.

Example

Describe the pattern. Then write the next three numbers. 4, 9, 14, 19, 24, 29

- | | |
|--|---|
| Step 1 Determine how the first number is changed to get to the second number. | To get to the second number you add 5 to the first number. $4 + 5 = 9$ |
| Step 2 Determine how the second number is changed to get to the third number. | To get to the third number you add 5 to the second number. $9 + 5 = 14$ |
| Step 3 Check the next two numbers to see if the pattern continues. | To get to the fourth number you add 5 to the third number. The same is true to get to the fifth number. |
| Step 4 Use the pattern to finish the sequence. | The next three numbers are 34, 39, 44. |

Practice

Describe the pattern. Then write the next three numbers.

1. 3, 5, 7, 9

Determine how the first number is changed to get to the second number.

You add _____ to the first number.

Determine how the second number is changed to get to the third number.

You add _____ to the second number.

Check the next two numbers to see if the pattern continues.

The pattern _____ continue.

Use the pattern to finish the sequence.

The last three numbers are _____.

2. 4, 8, 12, 16, 20, 24 _____

5. 45, 40, 35, 30, 25 _____

3. 11, 16, 21, 26, 31 _____

6. 22, 19, 16, 13, 10 _____

4. 5, 6, 8, 9, 11, 12 _____

7. 25, 21, 22, 18, 19, 15 _____

Number Patterns—Geometric Sequences

A useful skill in mathematics is to find a pattern in a series of numbers. When you look for a number pattern, you check to see how the numbers change from one to the next. For example, look at the following number sequence: 2, 4, 8, 16, 32, 64.

The pattern is to multiply each number by 2 to get to the next number. A number sequence in which the previous number is multiplied or divided by a certain number to get to the next number is a *geometric sequence*.

Determining the Rule in a Geometric Sequence

1. Determine how the first number is changed to get to the second number.
2. Determine how the second number is changed to get to the third number.
3. Check the next two numbers to see if the pattern continues.
4. Use the pattern to finish the sequence if needed.

Example

Describe the pattern. Then write the next three numbers. 4, 12, 36, 108

- | | |
|--|--|
| Step 1 Determine how the first number is changed to get to the second number. | To get to the second number you multiply the first number by 3. $4 \times 3 = 12$ |
| Step 2 Determine how the second number is changed to get to the third number. | To get to the third number you multiply the second by 3. $12 \times 3 = 36$ |
| Step 3 Check the next two numbers to see if the pattern continues. | To get to the fourth number you multiply the third number by 3. The same is true to get to the fifth number. |
| Step 4 Use the pattern to finish the sequence. | $108 \times 3 = 324$, $324 \times 3 = 972$, $972 \times 3 = 2,916$ |

Practice

Describe the pattern, and then write the next three numbers.

1. 64, 32, 16, 8

Determine how the first number is changed to get to the second number.

You _____ the first number by 2.

Determine how the second number is changed to get to the third number.

You _____ the second number by 2.

Check the next two numbers to see if the pattern continues.

The pattern _____ continue.

Use the pattern to finish the sequence.

$8 \div 2 = \underline{\hspace{1cm}}$, _____,

2. 2, 6, 18, 54 _____

3. 3, 4.5, 6.75 _____

4. 96, 48, 24, 12 _____

Estimation—Rounding

Rounding means changing a number to the nearest specific place value, such as tens, hundreds, thousands, and so on.

Rules for Rounding

1. Find the digit in place value to be rounded. Underline that digit.
2. Look at the digit to the right of the underlined digit.
If the digit to the right is 5 or more, add 1 to the digit in the rounding place.
If the digit to the right is less than 5, leave the rounding digit alone.
3. Change each digit to the right of the rounding digit to 0.

Example

Round 2,468 to the nearest hundred.

Step 1 Find the digit in the rounding place— underline that digit. 2,468

Step 2 Look at the digit to the right of the underlined digit. The digit to the right of the rounding digit is 6. 6 is greater than 5. So you increase the rounding digit by 1 to 5.
If the digit to the right is 5 or higher, add 1 to the rounding digit.
If the digit to the right is less than 5, then leave the rounding digit alone.

Step 3 Change each digit to the right of the rounding digit to 0. 2,468 → 2,500

Practice

Round each number to the specified place.

1. 12,277 to the nearest thousand

Find the digit in the rounding place— underline that digit. _____

Look at the digit to the right of the underlined digit. The number to the right of the rounding digit is _____, which is _____ than 5.
If the digit to the right is 5 or higher, add 1 to the rounding digit. So you _____.
If the digit to the right is less than 5, then leave the rounding digit alone.

Change each digit to the right of the rounding digit to 0. 12,277 → _____

- | | |
|---------------------------------------|---|
| 2. 4,763 to the nearest ten _____ | 6. 5,162 to the nearest thousand _____ |
| 3. 259 to the nearest hundred _____ | 7. 19,262 to the nearest thousand _____ |
| 4. 1,484 to the nearest hundred _____ | 8. 45,465 to the nearest ten thousand _____ |
| 5. 9,444 to the nearest hundred _____ | 9. 18,799 to the nearest hundred _____ |