Name ______________________________ Date ______________

Squaring a Binomial

When you square a binomial, you can apply the FOIL method to find the product. You can also apply the following rules as a short cut.

Rules for Squaring a Binomial
1. Square the first term.
2. Find 2 times the product of the two terms; use the same operation sign as the one between the two terms.
3. Square the last term.

Example

Solve. $(x + 3)^2$

Step 1 Square the first term.

x is the first term

$(x \times x) = x^2$

Step 2 Find 2 times the product of the two terms; use the same operation sign as the one between the two terms.

$2(x \times 3) = 2(3x) = 6x$

Use the plus sign.

Step 3 Square the last term.

$3^2 = 9$

$(x + 3)^2 = x^2 + 6x + 9$

Practice

Solve.

1. $(5x - 2)^2$

   Square the first term.

   $5x$ is the first term

   $(5x \times 5x) =$ ____________

   Find 2 times the product of the two terms; use the same operation sign as the one between the two terms.

   $2(5x \times 2) = 2(10x) =$ ________

   Use the ____________ sign.

   Square the last term.

   $-2^2 = 4$

   $(5x - 2)^2 =$ ____________

2. $(x + 4)^2$ ______________________________
3. $(x - 8)^2$ ______________________________
4. $(2x + 6)^2$ ______________________________
5. $(4x - 4)^2$ ______________________________
6. $(6x + 12)^2$ ______________________________

Name ______________________________ Date ______________________

Adding Polynomials

When you add two polynomials, you do so by adding like terms. Terms are like terms if they have the same variable raised to the same power.

Like Terms	Not Like Terms
$2x^2, -4x^2$	$5x^4, 4x^5$

Rules for Adding Polynomials

1. Write each polynomial in standard form.
2. Line up like terms.
3. Add the numbers in front of each variable. (Remember "1" is understood to be in front of a variable with no number).

Example

Add. $(4x^2 + 2x - 5) + (3x^4 - 3x + 5x^2)$

Step 1 Write each polynomial in standard form. $(4x^2 + 2x - 5) + (3x^4 + 5x^2 - 3x)$

Step 2 Line up like terms.

$$\begin{array}{r} 4x^2 + 2x - 5 \\ 3x^4 + 5x^2 - 3x \phantom{{}-5} \end{array}$$

Step 3 Add the numbers in front of each variable.

$$\begin{array}{r} 4x^2 + 2x - 5 \\ \underline{+3x^4 + 5x^2 - 3x \phantom{{}-5}} \\ 3x^4 + 9x^2 - 1x - 5 \end{array}$$

Practice

Add.

1. $(16x^3 + 5 - 2x^2) + (5 + 3x^3 + x^2)$

Write each polynomial in standard form. $(16x^3 - 2x^2 + 5) +$ ______________

Line up like terms. $16x^3 - 2x^2 + 5$

____

Add the numbers in front of each variable. $16x^3 - 2x^2 + 5$

+______________

____

2. $(-2x^3 + 5 - 3x) + (2x^3 + 2x - 3x^2)$ ______________________________

3. $(15 + 4x^2 + x^3) + (5x^3 - 10 - x^2)$ ______________________________

4. $(5xy + 3 - 2x^2y) + (14 + 5x^2y + xy)$ ______________________________

5. $(7 - 5y^3 + 3y^4) + (2y^2 - y^4 + 12y^3)$ ______________________________

Name ______________________________ Date ______________

Subtracting Polynomials

When you subtract two polynomials, you do so by subtracting like terms. Terms are like terms if they have the same variable raised to the same power.

Like Terms	Not Like Terms
$3x^3$, $-x^3$	$5x$, x^5

Rules for Subtracting Polynomials

1. Write polynomials in standard form.
2. Line up like terms.
3. Change the sign of each term in the second polynomial.
4. Add the numbers in front of each variable. (Remember, "1" is understood to be in front of a variable with no number).

Example

Subtract. $(2x^2 + 5x^3 + 1) - (5 + 5x^2 + 3x^3)$

Step 1 Write polynomials in standard form. $(5x^3 + 2x^2 + 1) - (3x^3 + 5x^2 + 5)$

Step 2 Line up like terms.

$$\begin{array}{l} 5x^3 + 2x^2 + 1 \\ -(3x^3 + 5x^2 + 5) \end{array}$$

Step 3 Change the sign of each term in the second polynomial.

$$\begin{array}{l} 5x^3 + 2x^2 + 1 \\ -3x^3 - 5x^2 - 5 \end{array}$$

Step 4 Add the numbers in front of each variable

$$\begin{array}{r} 5x^3 + 2x^2 + 1 \\ \underline{+ (-3x^3 - 5x^2 - 5)} \\ 2x^3 - 3x^2 - 4 \end{array}$$

Practice

Subtract.

1. $(x + 5 + 4x^2) - (10 + 3x^2 - 5x)$

Write polynomials in standard form. $(4x^2 + x + 5) -$ ____________

Line up like terms.

$4x^2 + x + 5$

$-$ ____________

Change the sign of each term in the second polynomial.

$4x^2 + x + 5$

____

Add the numbers in front of each variable

$4x^2 + x + 5$

$+$ ____________

____

2. $(x^2 + 5 - 4x^3) - (-10 - 2x^2 + 5x^3)$ ______________________

3. $(-4 + x^3 - 3x^2) - (5x + 5x^2 + 14)$ ______________________

4. $(-3x^2 + 10 - 2x) - (9 + 4x^2 - 10x)$ ______________________

5. $(-4x^3 + 12 - x) - (-5x + 10x^2 - 3)$ ______________________

Name ______________________________ Date ____________________

Multiplying a Polynomial

When you multiply a polynomial by a monomial, you multiply each term of the polynomial by the monomial. When you multiply one term by another, you multiply the numbers in front of each term. You also apply the rules for multiplying exponents by adding exponents.

$$(5x^2)(2x^3) = 10x^5 \qquad (2x^3)(3xy) = 6x^4y$$

Rules for Multiplying a Polynomial by a Monomial
Multiply each term in the polynomial by the monomial:
1. Multiply the numbers in front of the variables. (Remember that "1" is understood to be in front of a variable with no number)
2. Add the exponents of variables of the same letter (Remember that "1" is understood to be the exponent of a variable with no exponent.)

Example

Multiply. $4x(2x^2 + x - 6)$

Step 1 Multiply each term in the polynomial by the monomial.

$(4x)(2x^2) = 8x^3$

$(4x)(x) = 4x^2$

$(4x)(-6) = -24x$

$8x^3 + 4x^2 - 24x$

Practice

Multiply.

1. $3x^3(4x^2 + 2x - 1)$

   Multiply each term in the polynomial by the monomial.

   $(3x^3)(4x^2) =$ ____________

   $(3x^3)(2x) =$ ____________

   $(3x^3)(-1) =$ ____________

   ____

2. $7x^2(-3x^3 + 2)$ ______________________
3. $-2x^4(-4x^2 + x - 3)$ ______________________
4. $x^2(3x^5 + 6x^3 + x)$ ______________________
5. $5x^4(-4x^4 - 2x^3 + 5)$ ______________________

Name ______________________________ Date ____________________

Factoring a Binomial

By applying the Distributive Property in reverse, you can factor out a common factor.

$$20 + 15 = (5 \times 4) + (5 \times 3) = 5\,(4 + 3)$$

Rules for Factoring Out the Greatest Common Factor: Factoring Binomials

1. Find the greatest common factor of all the terms.
2. Determine the terms that when multiplied by the greatest common factor will result in each original term.
3. Rewrite the expression with the greatest common factor outside the parentheses and the terms you found in Step 2 inside the parentheses.

Example

Factor out the greatest common factor: $5x^2 + 10x$

Step 1 Find the greatest common factor of all the terms.

List the factors of $5x^2$ and $10x$.
$5x^2$: 1, 5, x^2
$10x$: 1, 2, 5, 10, x
The greatest common factor is $5x$.

Step 2 Determine the terms that when multiplied by the greatest common factor will result in each original term.

$(5x)\,(x) = 5x^2$
$(5x)\,(2) = 10x$

Step 3 Rewrite the expression with the greatest common factor outside the parentheses and the terms you found in Step 2 inside the parentheses.

$5x\,(x + 2)$

Practice

Factor out the greatest common factor.

1. $10x^4 - 15x^3$

Find the greatest common factor of all the terms.

List all the factors of $10x^4$ and $-15x^3$.
$10x^4$: 1, 2, 5, 10, x^4
$-15x^3$: 1, 3, 5, 15, x^3
The greatest common factor is ________ .

Determine the terms that when multiplied by the greatest common factor will result in each original term.

(________)(________) = $10x^4$
(________)(________) = $-15x^3$

Rewrite the expression with the greatest common factor outside the parentheses and the terms you found in Step 2 inside the parentheses.

________(______________)

2. $27x^4 - 9x^5$ __________ **3.** $36x^3 + 24x$ __________ **4.** $5x^2y^5 + 15xy^7$ __________

Name ________________________________ Date ____________________

Finding the Greatest Common Factor for Variable Terms

The greatest common factor can also be found for two or more variable terms. To find the greatest common factor for variable terms, you apply the rules you have learned for finding the greatest common factor of two or more numbers.

Rules for Finding the Greatest Common Factor for Variable Terms

1. List all the factors of the coefficients (the number in front of the variable).
2. From the lists, identify the greatest common factor; this is the coefficient-part of the greatest common factor.
3. List the variables with their exponents.
4. From the list, select the variable with the lowest exponent.

Example

Find the greatest common factor of $30x^3$, $40x^6$, and $50x^7$.

Step 1	List all the factors of the coefficients (the number in front of the variable).	30: 1, 2, 3, 5, 6, <u>10</u>, 15, 30 40: 1, 2, 4, 5, 8, <u>10</u>, 20, 40 50: 1, 2, 5, <u>10</u>, 25, 50
Step 2	From the list, identify the greatest common factor; this is the coefficient-part of the greatest common factor.	Of the factors listed, 10 is the greatest common factor.
Step 3	List the variables with their exponents.	The variable part of each term: x^3, x^6, x^7.
Step 4	From the list, select the variable with the lowest exponent.	The variable with the lowest exponent is x^3. The greatest common factor is $10x^3$.

Practice

Find the greatest common factor for each list of terms.

1. $12m^3n^2$ and $18m^5n^4$

List all the factors of the coefficients (the number in front of the variable).	12: 1, 2, 3, 4, 6, 12 18: ______
From the list, identify the greatest common factor; this is the coefficient-part of the greatest common factor.	Of the factors listed, ____ is the greatest common factor.
List the variables with their exponents.	$m^3n^2 = m^3 \times n^2$ $\quad$ $m^5n^4 =$ ____ $\times$ ____
From the list, select the variable with the lowest exponent.	The variable with the lowest exponents are m^3 and ____. The greatest common factor is ____ m^3____

2. $6x^4$ and $8x^6$ ____________

3. $9m^2$ and $3m^5$ ____________

4. $12x^8$ and $20x^9$ ____________

5. $4x^4y^3$ and $6xy^2$ ____________

Name ____________________ Date ____________________

Factoring a Polynomial

By applying the Distributive Property in reverse, you can factor out a common factor.

$$20 + 15 = (5 \times 4) + (5 \times 3) = 5\,(4 + 3)$$

Rules for Factoring Out the Greatest Common Factor: Factoring Polynomials

1. Find the greatest common factor for all of the terms.
2. Determine the terms that when multiplied by the greatest common factor will result in each original term.
3. Rewrite the expression with the greatest common factor outside the parentheses and the terms found in Step 2 inside the parentheses.

Example

Factor out the greatest common factor. $20x^5 + 10x^6 - 15x^4$

Step 1 Find the greatest common factor for all of the terms.

List the factors of $20x^5$, $10x^6$, and $-15x^4$.
$20x^5 = 1, 2, 4, 5, 10, 20, x^5$
$10x^6 = 1, 2, 5, 10, x^6$
$-15x^4 = 1, 3, 5, 15, x^4$
The greatest common factor is $5x^4$.

Step 2 Determine the terms that when multiplied by the greatest common factor will result in each original term.

$(5x^4)\,(4x) = 20x^5$;
$(5x^4)(2x^2) = 10x^6$
$(5x^4)(-3) = -15x^4$

Step 3 Rewrite the expression with the greatest common factor outside the parentheses and the terms found in Step 2 inside the parentheses.

$5x^4\,(4x + 2x^2 - 3)$

Practice

Factor out the greatest common factor.

1. $13x^8 + 26x^4 - 39x^2$

Find the greatest common factor for all of the terms.

List all the factors of $13x^8$, $26x^4$, and $-39x^2$.
$13x^8$: ____________
$26x^4$: ____________
$-39x^2$: ____________
The greatest common factor is $13x^2$.

Determine the terms that when multiplied by the greatest common factor will result in each original term.

$13x^2\,(x^6) = 13x^8$ $13x^2$ (________) $= 26x^4$
$13x^2$ (____________) $= -39x^2$

Rewrite the expression with the greatest common factor outside the parentheses and the terms found in Step 2 inside the parentheses.

$13x^2\,(x^6$ ________)

2. $8x^3 - 24x^2 - 80x$ ____________________

3. $3x^4 + 30x^3 + 48x^2$ ____________________

4. $x^2 + 2x^5 - 9x^7$ ____________________

5. $3x^5 - 6x^4y - 45x^3y^2$ ____________________

Name ______________________________ Date ______________

Factoring Trinomials in the Form $x^2 + bx + c$

You can also factor a polynomial. When you factor a polynomial you look for pairs of expressions whose product (when they are multiplied) is the original polynomial.

$$(x + 2)\,(x + 4) = x^2 + 6x + 8$$

Rules for Factoring a Trinomial in the Form $x^2 + bx + c$

1. Create a table. The left column lists the factors of c. The right column is the sum of the factors in column 1.
2. Choose the pair of factors in the right column whose sum equals b.
3. Create two expressions of "$x +$" the factors.

Example

Factor: $x^2 + 9x + 18$

Step 1 Create a table. The left column lists the factors of c. The right column is the sum of the factors in column 1.

FACTORS OF 18	SUM OF FACTORS
1 and 18	1 + 18 = 19
2 and 9	2 + 9 = 11
3 and 6	3 + 6 = 9

Step 2 Choose the pair of factors in the right column whose sum equals b.

The last pair of factors, 3 and 6, have a sum (9) that equals the value of b.

Step 3 Create two expressions of "$x +$" the factors.

The factors are: $(x + 3)(x + 6)$.

Practice

Factor.

1. $x^2 - 8x + 12$

Create a table. The left column lists the factors of c. The right column is the sum of the factors in column 1.

Note that since b is negative, we are using negative numbers for the factors of c.

FACTORS OF 12	SUM OF FACTORS
–1 and –12	–1 + –12 = –13
–2 and –6	–2 + –6 = –8
–3 and –4	–3 + –4 = –7

Choose the pair of factors in the right column whose sum equals b.

The pair of factors ______ and ______ have a sum that equals the value of b, ______.

Create two expressions of "$x +$" the factors.

The factors are:
$(x + (-2))(x + (-6))$ or $(x - 2)(x - 6)$

2. $x^2 + 4x + 3$ ______________________

3. $x^2 + 9x + 8$ ______________________

4. $x^2 + 8x + 15$ ______________________

5. $x^2 - 15x + 36$ ______________________

Name ______________________________ Date ________________

Factoring Trinomials in the Form $ax^2 + bx + c$

You can use FOIL to multiply binomials, creating a trinomial. You can also use FOIL to "undo" a trinomial, creating two binomials.

> **Rules for Factoring Trinomials in the Form $ax^2 + bx + c$**
> 1. Create a FOIL table.
> 2. In the "F" column place factors that result in *a*. In the "L" column place factors that result in *c*.
> 3. In the "O" and "I "columns, try different combinations of the factors from Step 2 by adding the products. The combination resulting in *b* shows the placement within the binomials.

Example

Factor. $6x^2 + 23x + 7$

Step 1 Create a FOIL table.

Step 2 In the "F" column, place factors that result in *a*. In the "L" column, place factors that result in *c*.

Step 3 In the "O + I" column, try different combinations of the factors from Step 2 by adding the products. The combination resulting in *b* shows the placement within the binomials.

$6x^2$	$23x$					7
F	O	+	I	=	?	L
1 × 6	1 × 7	+	1 × 6	=	13	1 × 7
	1 × 1	+	7 × 6	=	43	
2 × 3	2 × 7	+	1 × 3	=	17	1 × 7
	2 × 1	+	7 × 3	=	23	

Value for *b*

The outer terms are 2 and 1. The inner terms are 7 and 3.

$6x^2 + 23x + 7 = (2x + 7)(3x + 1)$

Practice

Factor.

1. $5x^2 - 14x - 3$

   Create a FOIL table.

   In the "F" column, place factors that result in *a*. In the "L" column, place factors that result in *c*.

   In the "O + I" column, try different combinations of the factors from Step 2 by adding the products. The combination resulting in *b* shows the placement within the binomials.

Value for *b*

$5x^2$	$-14x$					-3
F	O	+	I	=	?	L
5 × 1	5 × 1	+	______	=	2	1 × (–3)
	______	+	1 × 1	=	____	
______	5 × (–1)	+	______	=	____	______
	5 × (3)	+	(–1) × 1	=	____	

$5x^2 - 14x - 3 = (\_\_\_\_x + \_\_\_\_)(\_\_\_\_x + \_\_\_\_)$

$= \_\_\_\_\_\_\_\_\_\_\_\_$

2. $2x^2 + 8x + 8$ ______________________
3. $2x^2 - 3x - 9$ ______________________
4. $7x^2 + 50x + 7$ ______________________
5. $2x^2 - 8x + 6$ ______________________
6. $2x^2 - 7x - 4$ ______________________

Name ______________________________ Date ____________________

The Difference of Two Squares

The difference of squares involves multiplying two binomials with the same two terms. One binomial is the sum of the terms—for example, $(a^2 + b)$. The other binomial is the difference of the terms—for example, $(a^2 - b)$. As you can see, the two terms are a^2 and b. When you multiply the two squares, the product follows a pattern.

Rules for the Difference of Squares
1. Square the first term.
2. Square the second term.
3. Place a minus sign between the two squared terms.

Example

Solve. $(x^2 + 4)(x^2 - 4)$

Step 1 Square the first term.

$(x^2)^2 = (x^2)(x^2) = x^4$

Remember when you multiply terms with exponents, you add the exponents.

Step 2 Square the second term.

$(4)^2 = 4 \times 4 = 16$

Step 3 Place a minus sign between the two squared terms.

Result of the first squaring: x^4

Result of the second squaring: 16

$x^4 - 16$

Practice

Solve.

1. $(x^4 + y)(x^4 - y)$

Square the first term. $(x^4)^2 = (x^4)(x^4) =$ ________

Square the second term. $(y)^2 = (y)(y) =$ ________

Place a minus sign between the two squared terms.

The result of the first squaring is ______ .

The result of the second squaring is ______ .

________ – ________

2. $(x^3 + y^2)(x^3 - y^2)$ ______________________________

3. $(x^5 + 5)(x^5 - 5)$ ______________________________

4. $(2x^3 + 4)(2x^3 - 4)$ ______________________________

5. $(4x^2 + 3)(4x^2 - 3)$ ______________________________

6. $(3x^4 + y^2)(3x^4 - y^2)$ ______________________________