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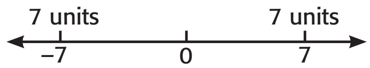
Absolute Value

The **absolute value** of a number is the distance between the origin of a number line and the point representing that number.

Look at the number line below. Both 7 and −7 are 7 units from the origin.



The notation for absolute value is $|a|$ and is read as "the absolute value of *a*."

Rules for the Absolute Value of a Number
1. If a is a positive number, then $|a| = a$ (e.g. $|4| = 4$).
2. If a is zero, then $|a| = 0$ (e.g. $|0| = 0$).
3. If a is a negative number, then $|-a| = a$ (e.g. $|-4| = 4$).

Example

Solve the equation. $|x| = 15$

Step 1 Which positive number is 15 units from the origin? 15

Step 2 Which negative number is 15 units from the origin? −15

Step 3 Check that both solutions are true. $|15| = 15$ true
$|-15| = 15$ true

Practice

Solve.

1. $-|x| = -10$

Which positive number is 10 units from the origin? ______

Which negative number is 10 units from the origin? ______

Check that both solutions are true. $-|$______$| = -10$ ______

$-|$______$| =$ ______ ______

2. $|x| = 3$ ____________

3. $|x| = 0$ ____________

4. $|-7| = x$ ____________

5. $|-x| = 13$ ____________

6. $-|5| = x$ ____________

7. $-|-6| = x$ ____________

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Absolute Value Equations

The **absolute value** of a number is the distance between the origin of a number line and the point representing that number. To solve an absolute value equation you need to account for the value inside the absolute value symbol being positive, and the value inside the absolute value symbol being negative.

Rules for Solving an Absolute Value Equation

1. Account for the value of the expression inside the absolute value symbol being positive.
2. Account for the value of the expression inside the absolute value symbol being negative.

Example

Solve. $|x + 2| = 10$

Step 1 Account for the value of the expression inside the absolute value symbol being positive.

What value of x will result in an amount of 10? 8

$|x + 2| = 10$

$|8 + 2| = 10$

$|10| = 10$ True

Step 2 Account for the value of the expression inside the absolute value symbol being negative.

What value of x will result in an answer of -10? -12

$|x + 2| = 10$

$|-12 + 2| = 10$

$|-10| = 10$ True

The solution is -12 and 8.

Practice

Solve.

1. $|2x - 1| = 9$

Account for the value of the expression inside the absolute value symbol being positive.

What value of x will result in an answer of 9?

_____

$|2x - 1| = 9$

$|2$ _____ $- 1| = 9$

$|$ _____ $| = 9$

Account for the value of the expression inside the absolute value symbol being negative.

What value of x will result in a value of -9?

_____

$|2x - 1| = 9$

$|2$ _____ $-1| = 9$

$|$ _____ $| = 9$

2. $|x - 4| = 8$ ____________________

3. $|x + 3| = 15$ ____________________

4. $|3x| = 21$ ____________________

5. $|3x + 3| = 30$ ____________________

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Compound Inequalities

A compound inequality is made of two inequalities connected by "and" or "or."

A compound inequality with "and": $-4 < x < 10$

A compound inequality with "or": $x < -2$ or $x > 5$

Rules for Solving a Compound Inequality
Compound inequality with "and" (x is in the middle of the expression).
1. Write the original inequality as two inequalities.
2. Solve for the left side of the inequality.
3. Solve for the right side of the inequality.
Compound inequality with "or."
1. Write the original inequality.
2. Solve the left side of the inequality.
3. Solve the right side of the inequality.

Example

Solve. $-4 \leq 2x < 10$ (Compound inequality with "and.")

Step 1	Write the original inequality as two inequalities.	$-4 \leq 2x < 10$ $-4 \leq 2x$ and $2x < 10$
Step 2	Solve for the left side of the inequality.	$-4 \leq 2x \rightarrow -2 \leq x$
Step 3	Solve for the right side of the inequality.	$2x < 10 \rightarrow x < 5$ $-2 \leq x < 5$

Practice

Solve.

1. $3x < 6$ or $2x + 2 > 10$ (Compound inequality with "or.")

Write the original inequality.	$3x < 6$ or $2x + 2 > 10$
Solve the left side of the inequality.	$3x < 6 \rightarrow$ ________
Solve the right side of the inequality.	$2x + 2 > 10 \rightarrow$ ________
	The solution is ________________.

2. $6x + 2 < -10$ or $4x > 16$ ______________________________

3. $-3x + 3 > 12$ or $4x > 4$ ______________________________

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Absolute Value Inequalities

When solving an absolute inequality you can apply what you know from solving absolute value equations. As with absolute value equations, you look for values that are positive and negative that make the inequality true.

Rules for Solving Absolute Value Inequalities

1. Write the absolute value inequality as an inequality with the original number to the right of the inequality symbol.
2. Write the absolute value inequality as an inequality with the inequality symbol reversed and the inverse of the number to the right of the symbol.
3. Solve each inequality. Write the solutions as a compound inequality.

Example

Solve. $|x + 2| > 10$

Step 1 Write the absolute value inequality as an inequality with the original number to the right of the inequality symbol.

$|x + 2| > 10$

$x + 2 > 10$

Step 2 Write the absolute value inequality as an inequality with the inequality symbol reversed and the inverse of the number to the right of the symbol.

$|x + 2| > 10$

$x + 2 < -10$

Step 3 Solve each inequality. Write the solutions as a compound inequality.

$x + 2 > 10 \rightarrow x > 8$

$x + 2 < -10 \rightarrow x < -12$

$x < -12$ or $x > 8$

Practice

Solve.

1. $|2x + 4| < 12$

Write the absolute value inequality as an inequality with the original number to the right of the inequality symbol.

$|2x + 4| < 12$

____

Write the absolute value inequality as an inequality with the inequality symbol reversed and the inverse of the number to the right of the symbol.

$|2x + 4| < 12$

____

Solve each inequality. Write the solutions as a compound inequality.

______________ $\rightarrow x <$ ______

______________ $\rightarrow x >$ ______

The solution is ______________.

2. $|x - 3| \geq 3$ ______________________

3. $|x + 5| \leq 10$ ______________________

4. $|5x - 15| > 5$ ______________________

5. $|x + 1| + 3 > 5$ ______________________

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Graphing Absolute Value Inequalities

The **absolute value** of a number is the distance between the origin of a number line and the point representing that number. When graphing an absolute value inequality, you first have to solve the inequality, treating it as a compound inequality. Then graph each solution.

> **Rules for Graphing an Absolute Value Inequality**
> 1. Rewrite the inequality as two inequalities.
> 2. Solve each inequality.
> 3. Graph each solution. If the solution is connected by "or," then the graph is away from the two points. If the solution is connected by "and," then the graph is between the two points.

Example

Solve, then graph the solution. $|x + 6| < 11$

Step 1 Rewrite the inequality as two inequalities. $x + 6 < 11$ and $x + 6 > -11$

Step 2 Solve each inequality. $x < 5$ and $x > -17$

Step 3 Graph each solution. If the solution is connected by "or," then the graph is away from the two points. If the solution is connected by "and," then the graph is between the two points.

Practice

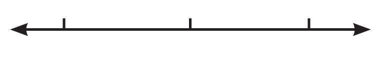
Solve, then graph the solution.

1. $|x + 5| \geq 3$

Rewrite the inequality as two inequalities. $x + 5 \geq 3$ or ____________

Solve each inequality. $x \geq$ ______ or $x \leq$ ______

Graph each solution. If the solution is connected by "or," then the graph is away from the two points. If the solution is connected by "and," then the graph is between the two points.



2. $|x + 4| > 5$ ____________

3. $|x + 7| \leq 10$ ____________

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Introduction to Matrices

A **matrix** is a rectangular array of numbers written within brackets. A matrix is identified by a capital letter. A matrix is classified by its dimensions—the number of columns and rows it contains.

Matrix X to the right has 3 rows and 2 columns. It is a 3×2 matrix.

A **matrix element** is a number in the matrix. Each matrix element is identified by its location within the matrix.

element X_{12}

$$X = \begin{bmatrix} 29{,}300 & 2{,}900 \\ 23{,}200 & 2{,}100 \\ 15{,}400 & 1{,}200 \end{bmatrix} \text{ 3 rows}$$

2 columns

> **Rules for Reading a Matrix**
> 1. The dimensions of a matrix are given in terms of rows and columns.
> 2. A matrix element is identified by (1) using the letter of the matrix, and (2) using a subscript to identify the position of the element by row and column.

Example

State the dimensions of the matrix. Identify element A_{23}. $A = \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 2 \end{bmatrix}$

Step 1 The dimensions of a matrix are given in terms of rows and columns.

The matrix has 2 rows and 3 columns; it is a 2×3 matrix.

Step 2 A matrix element is identified by (1) using the letter of the matrix, and (2) using a subscript to identify the position of the element by the row and column.

A_{23} is the element in row 2, column 3. $A_{23} = 2$

Practice

State the dimensions of the matrix. Identify the specified element.

1. Identify element B_{22}.

$B = \begin{bmatrix} 3 & 9 & 1 & 6 \\ 0 & 7 & 9 & 7 \end{bmatrix}$

The dimensions of a matrix are given in terms of rows and columns.

The matrix has _____ rows and _____ columns; it is a _______ matrix.

A matrix element is identified by (1) using the letter of the matrix, and (2) using a subscript to identify the position of the element by the row and column.

B_{22} is the element in row _____, column _____. B_{22} = _____

2. Identify element Z_{21}. $Z = \begin{bmatrix} 10 & 0 \\ -2 & 1 \end{bmatrix}$ ____________________

3. Identify the location of –10. $Z = \begin{bmatrix} 0 & -1 & -4 & 5 \\ 3 & 5 & -10 & 7 \\ 6 & -3 & -1 & 0 \end{bmatrix}$ ____________________

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Matrix Addition

When adding matrices, you add the corresponding elements in each matrix.

corresponding elements

$$\begin{bmatrix} -2 & 0 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -4 & 7 \end{bmatrix}$$

> **Rule for Matrix Addition**
> Add corresponding elements in each matrix to form one large matrix.

Example

Add. $\begin{bmatrix} -4 & 2 \\ -10 & 7 \end{bmatrix} + \begin{bmatrix} 5 & -9 \\ 9 & -3 \end{bmatrix}$

Add corresponding elements in each matrix to form one large matrix.

$$\begin{bmatrix} 4 & 2 \\ -10 & 7 \end{bmatrix} + \begin{bmatrix} 5 & -9 \\ 9 & -3 \end{bmatrix} = \begin{bmatrix} 4+5 & 2+(-9) \\ (-10)+9 & 7+(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -7 \\ -1 & 4 \end{bmatrix}$$

Practice

Add.

1. $\begin{bmatrix} -5 & 8 \\ 3 & -3 \end{bmatrix} \quad \begin{bmatrix} 11 & -1 \\ 5 & -7 \end{bmatrix}$

Add corresponding elements in each matrix to form one large matrix.

$$\begin{bmatrix} -5 & 8 \\ 3 & -3 \end{bmatrix} + \begin{bmatrix} 11 & -1 \\ 5 & -7 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

$$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

2. $\begin{bmatrix} 2 & -9 & -4 \\ 3 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 16 & 9 \\ 11 & 1 & 2 \end{bmatrix}$ ______________________________

3. $\begin{bmatrix} -4 & 7 \\ -9 & 3 \end{bmatrix} + \begin{bmatrix} 7 & 2 \\ -4 & 20 \end{bmatrix}$ ______________________________

4. $\begin{bmatrix} 2 \\ 17 \end{bmatrix} + \begin{bmatrix} 5 \\ -12 \end{bmatrix}$ ______________________________

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Matrix Subtraction

When subtracting matrices, you subtract the corresponding elements in each matrix.

$$\begin{bmatrix} \overset{\text{corresponding elements}}{\downarrow}\!\!-2 & 0 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} \downarrow 3 & -1 \\ -4 & 7 \end{bmatrix}$$

Rule for Matrix Subtraction
Subtract corresponding elements in each matrix to form one large matrix.

Example
Subtract. $\begin{bmatrix} -2 & 5 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} -4 & 6 \\ 8 & 5 \end{bmatrix}$

Subtract corresponding elements in each matrix to form one large matrix.

$$\begin{bmatrix} -2 & 5 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} -4 & 6 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} -2-(-4) & 5-6 \\ 0-8 & -2-5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -8 & -7 \end{bmatrix}$$

Practice

Subtract.

1. $\begin{bmatrix} 3 & 3 \\ -4 & -1 \end{bmatrix} - \begin{bmatrix} 6 & -2 \\ 8 & -2 \end{bmatrix}$

Subtract corresponding elements in each matrix to form one large matrix.

$$\begin{bmatrix} 3 & 3 \\ -4 & -1 \end{bmatrix} - \begin{bmatrix} 6 & -2 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

$$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

2. $\begin{bmatrix} -3 & 5 \\ 0 & -4 \end{bmatrix} - \begin{bmatrix} -5 & 9 \\ 10 & 3 \end{bmatrix}$ ____________________

3. $\begin{bmatrix} 9 & -12 & 15 \\ 4 & 7 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 4 & -3 \\ 5 & -1 & -3 \end{bmatrix}$ ____________________

4. $\begin{bmatrix} 6 & 12 \\ -8 & -4 \end{bmatrix} - \begin{bmatrix} -9 & 0 \\ -2 & -9 \end{bmatrix}$ ____________________

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Scalar Multiplication

A **matrix** is a rectangular arrangement of numbers in rows and columns. You can think of a matrix as a way to organize data, similar to the way data is displayed in a table. A **scalar** is a real number factor by which all the elements of a matrix are multiplied.

Rule for Scalar Multiplication
Create an expanded matrix by multiplying each element by the scalar.

Example

Solve. $2\begin{bmatrix} -6 & 4 \\ 7 & -3 \end{bmatrix}$

Create an expanded matrix by multiplying each element by the scalar.

$$2\begin{bmatrix} -6 & 4 \\ 7 & -3 \end{bmatrix} = \begin{bmatrix} -6 \times 2 & 4 \times 2 \\ 7 \times 2 & -3 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 8 \\ 14 & -6 \end{bmatrix}$$

Practice

Solve.

1. $5\begin{bmatrix} 11 & -9 & -4 \\ -5 & 6 & 3 \end{bmatrix}$

Create an expanded matrix by multiplying each element by the scalar.

$$5\begin{bmatrix} 11 & -9 & -4 \\ -5 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 11 \times 5 & \_\_\_\_ & \_\_\_\_ \\ -5 \times 5 & \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

$$= \begin{bmatrix} 55 & \_\_\_\_ & \_\_\_\_ \\ -25 & \_\_\_\_ & \_\_\_\_ \end{bmatrix}$$

2. $-3\begin{bmatrix} 2 & 16 \\ 9 & -2 \\ -11 & 6 \end{bmatrix}$ ______________________________

3. $4\begin{bmatrix} 5 & -12 \\ 8 & -2 \end{bmatrix}$ ______________________________

4. $-6\begin{bmatrix} -8 & -4 & 1 \\ 0 & 2 & -9 \end{bmatrix}$ ______________________________

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Matrix Multiplication

When multiplying matrices, you multiply the elements of a row in the first matrix by the corresponding elements in a column of the second matrix. You then add the products.

$$\begin{bmatrix} 3 & 6 & 5 \\ 5 & 3 & 8 \end{bmatrix} \times \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \times 5 & + & 6 \times 2 & + & 5 \times 2 \\ 5 \times 5 & + & 3 \times 2 & + & 8 \times 2 \end{bmatrix}$$

Rules for Matrix Multiplication

1. Circle each row of the first matrix; circle each column of the second matrix.

2. Multiply the elements of a row in the first matrix by the elements of each column in the second matrix. Add the products in each row.

The dimensions of the resulting matrix will be the number of rows in the first matrix by the number of columns in the second matrix.

Example

Multiply. $\begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 9 \end{bmatrix}$

Step 1 Identify the elements to be multiplied.

$$\begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

Step 2 Multiply the elements of a row in the first matrix by the elements of each column in the second matrix.

$$\begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \times 2 + 5 \times 9 \\ 2 \times 2 + -3 \times 9 \end{bmatrix}$$

Add the products in each row.

$$= \begin{bmatrix} 6 + 45 \\ 4 + -27 \end{bmatrix} = \begin{bmatrix} 51 \\ -23 \end{bmatrix}$$

Practice

Multiply.

1. $\begin{bmatrix} 2 & -4 \\ 3 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 9 \\ 3 & 2 \end{bmatrix}$

Identify the elements to be multiplied.

$$\begin{bmatrix} 2 & -4 \\ 3 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 9 \\ 3 & 2 \end{bmatrix}$$

Multiply the elements of a row in the first matrix by the elements of each column in the second matrix.

$$\begin{bmatrix} 2 & -4 \\ 3 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 9 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Add the products in each row.

$$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

2. $\begin{bmatrix} 6 & 3 & 8 \\ 9 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 7 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} \quad \\ \quad \end{bmatrix}$

3. $\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} \times \begin{bmatrix} 9 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$