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OVERVIEW

Early Numeracy is a curriculum designed to build number sense in elementary-aged students with disabilities, which includes students with moderate-to-severe intellectual disabilities and/or autism. The curriculum addresses the adaptations required for instruction of students who are nonverbal, have visual impairments, have physical limitations, or who are deaf. For example, students may use assistive technology, point, tap, or eye gaze to make responses. The curriculum was designed for students who lack a solid foundation, or need additional practice to build fluency with their early numeracy skills.

Early Numeracy provides strategies to improve these skills:

- Counting with one-to-one correspondence
- Numeral identification from 1 to 10
- Numeral naming 1–10
- Rote counting to 20
- Creating sets to 10
- Beginning addition of sets
- Identification and understanding of numeracy symbols
- Identification, extension, and creation of ABAB patterns
- Use and understanding of calendars
- Use of nonstandard and standard units of measurement.

The full scope and sequence is presented on pp. 7–8. The Early Numeracy curriculum is comprised of four units with six theme-based lessons per unit. Each lesson is scripted, making it teacher friendly and easy to use. Twelve early numeracy skills have been selected for this curriculum, and these twelve skills build in difficulty across each unit. The Early Numeracy curriculum is equipped with forms to help plan how skills will be embedded within a general education mathematics class setting to promote generalization.

Rationale for Content

Research shows that early knowledge in mathematics is indicative of success in mathematics in later years of life (Denton & West, 2002; Horne, 2005; National Mathematics Advisory Panel [NMP], 2008). For most students, these early numeracy skills are developed prior to any formal schooling as a result of their interactions and experiences; however, for students with moderate-to-severe disabilities, this may not be the case. This population of students needs formal instruction to help develop these skills (Bruer, 1997). Just as phonemic awareness builds the foundation of reading in very early grades, early numeracy skills build the foundation in mathematics.

The Early Numeracy curriculum is based on the premise that it is not too late to develop early numeracy skills in children with moderate-to-severe disabilities between the ages of 5 and 11. The content selected for the Early Numeracy curriculum was based primarily on recommendations from *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children*, developed by Sarama and Clements (2009). In adapting these skills for students with moderate-to-severe disabilities, feedback from national mathematics experts in elementary education and special education experts in severe disabilities was acquired. The Early Numeracy curriculum aligns with four of the five content standards set forth by the National Council of Teachers of Mathematics (NCTM): *Numbers and Operations*, *Algebra*, *Geometry*, and *Measurement* (NCTM, 2000).

Rationale for Method of Instruction

In a meta-analysis of the experimental research on teaching mathematics to students with significant cognitive disabilities,

Browder, Spooner, Ahlgrim-Dezell, Harris, and Wakeman (2008) found that systematic instruction—with specific prompt fading procedures such as least intrusive prompts and time delay with feedback—is an evidence-based practice for teaching mathematics to this population. In addition, they found strong support for the use of in-vivo instruction, where students applied their learning to real-world situations. Both the systematic instruction approach and in-vivo instruction are strongly supported through the quality and quantity of research studies and through the large effect sizes produced in these studies, showing that students with significant cognitive disabilities can learn mathematical skills through these methods of intervention. The Early Numeracy curriculum was developed using these evidence-based practices to help bridge gaps and build fluency in early numeracy skills.

The overarching goal of the Early Numeracy curriculum is to better prepare students to participate in general curriculum math lessons, or lessons aligned to grade-level mathematics content that require students to have mastery of basic numeracy skills. Students with

moderate-to-severe disabilities can learn mathematics skills aligned with grade-level content using adaptations (Browder, Jimenez, & Trela, 2012; Browder et al., 2010). The Early Numeracy curriculum can promote greater independence and conceptual understanding for this goal by teaching early numeracy skills to mastery. The developers of this curriculum strongly suggest that the Early Numeracy curriculum be used as a supplement to general education mathematics instruction so students are still receiving valuable opportunities to practice using these skills in other contexts. In literacy, students gain the foundation for learning to read through the *Early Literacy Skills Builder*, but also acquire grade-aligned skills through the read alouds of the literature of their grade level (Browder, Gibbs, Ahlgrim-Dezell, Courtade, & Lee, 2007). Students gain the foundation for learning mathematics through the Early Numeracy curriculum, but also acquire **grade-aligned skills** through doing hands-on problem solving and embedding skills in the general education mathematics lesson.

SCOPE AND SEQUENCE

There are four units of instruction, which address 7 domains, in the Early Numeracy curriculum. The following scope and sequence presents the domains, skills, and themes across the four units of the curriculum.

Domain	UNIT ONE Math Is Everywhere	UNIT TWO Math at Celebrations	UNIT THREE Math in Nature	UNIT FOUR Math + Me = Fun
Counting	1 Count 1–5 movable objects in a line.	1 Count out 1–5 movable objects from a group.	1 Count 1–10 movable objects in a line.	1 Count out 1–10 movable objects from a group.
	2 Count 1–5 nonmovable objects in a line.	2 Count 1–5 scattered, nonmovable objects.	2 Count 1–10 nonmovable objects in a line.	2 Count 1–10 scattered, nonmovable objects.
	3 Rote count from 1–5.	3 Rote count from 1–10.	3 Rote count from 1–15.	3 Rote count from 1–20.
Sets	4 Make sets of 1–3.	4 Make sets of 1–4.	4 Make sets of 1–9.	4 In context, make sets of 1–9.
	5 Add premade sets with sums to 5.	5 Add sets with sums to 5.	5 Add sets with sums to 10.	5 In context, add sets with sums to 10.
Symbol Use	6 Compare sets for same/equal.	6 Compare sets for greater than.	6 Compare sets for less than.	6 Compare sets and numbers for equal, greater than, and less than.
	7 Identify the symbol for equals (=).	7 Identify the symbol for greater than (>).	7 Identify the symbol for less than (<).	7 Use symbols for equals, greater than, and less than (=, >, <).

(Scope and Sequence continues)

(Scope and Sequence continued)

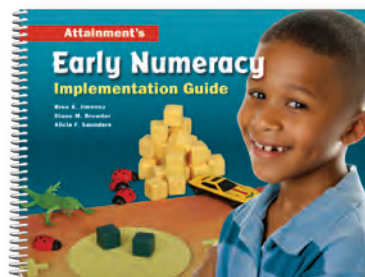
Domain	UNIT ONE Math Is Everywhere	UNIT TWO Math at Celebrations	UNIT THREE Math in Nature	UNIT FOUR Math + Me = Fun
Patterns	8 Identify an ABAB pattern.	8 Extend an ABAB pattern.	8 Create an ABAB pattern.	8 Complete an ABAB pattern with missing components.
Measurement	9 Use a nonstandard unit of measurement to measure 1–5.	9 Use a standard unit of measurement to measure 1–5 inches.	9 Use a standard unit of measurement to measure 1–10 inches.	9 Convert inches to feet.
Calendar	10 Identify dates from 1st to 5th on a calendar.	10 Identify dates from 1st to 10th on a calendar.	10 Name dates from 1st to 5th on a calendar.	10 Name dates from 1st to 10th on a calendar.
	11 Identify 1–5 days later in a week using a calendar.	11 Identify 1–5 days later across 2 weeks using a calendar.	11 Identify 1–10 days later across 2 weeks using a calendar.	11 Identify 1–10 days later across 3 weeks using a calendar.
Numeral Identification	12 Identify numerals 1–5.	12 Identify numerals 1–10.	12 Name numerals 1–5.	12 Name numerals 1–10.
Themes	Lesson 1 Math at the Speedway Lesson 2 Math Treasures Lesson 3 Gardening with Math Lesson 4 Beach Math Lesson 5 Math Class Trip Lesson 6 Soccer Review	Lesson 1 Mardi Gras Math Lesson 2 Math in the New Year Lesson 3 Math at the Fiesta Lesson 4 Math at the Family Feast Lesson 5 Going to a Pow Wow Lesson 6 Basketball Review	Lesson 1 Math in the Flower Garden Lesson 2 Backyard Buggy Math Lesson 3 Fishing for Numbers Lesson 4 Math at the Aquarium Lesson 5 Froggy Math Lesson 6 Football Review	Lesson 1 Butterfly Math Lesson 2 Math at the Ballgame Lesson 3 Math in the Desert Lesson 4 Math at the State Fair Lesson 5 Math in the Berry Patch Lesson 6 Baseball Review

EARLY NUMERACY MATERIALS

The Early Numeracy curriculum comes with everything you need to get started teaching early numeracy skills.

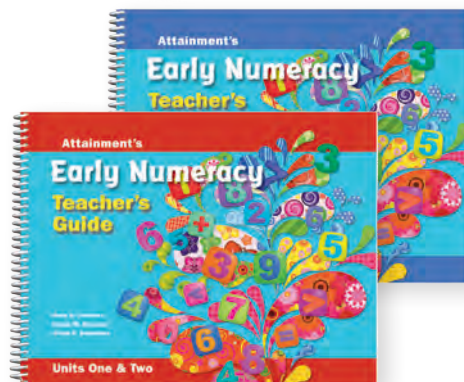
Implementation Guide

The Implementation Guide describes the Scope and Sequence, research outcomes, how to teach using the curriculum, how to determine a starting point, how to measure progress, and how to embed early numeracy skills in a general education math class. The Implementation Guide also includes reproducible forms and appendixes referred to in the curriculum.



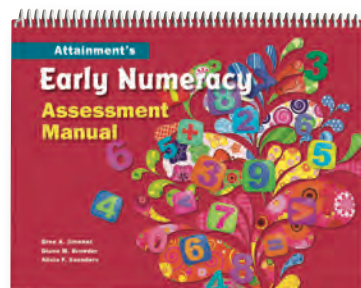
Teacher's Guides with Scripted Lessons

Two Teacher's Guides are included in the Early Numeracy curriculum, one for Units One and Two, the other for Units Three and Four. The Teacher's Guides provide the plans for teaching the lessons.



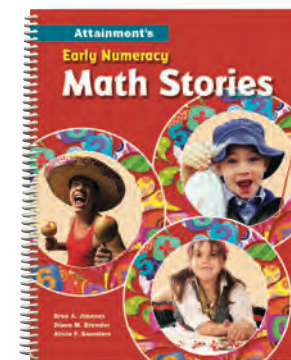
Assessment Manual

The Assessment Manual provides unit assessments that will help you determine where to start. The assessment by unit will also help you measure progress or mastery after unit completion.



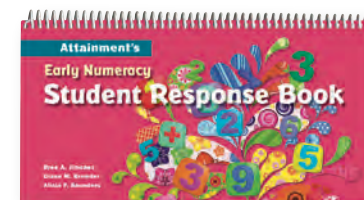
Math Stories

Math Stories are engaging stories to get students thinking about the math lesson.



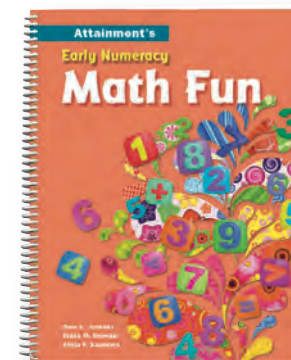
Student Response Book

The Student Response book is used during the lesson to give students multiple opportunities to practice the skill being learned.



Math Fun

Each student has their own math workbook, called Math Fun, for practicing the newly learned skills.



CD

The CD includes PDFs of books for convenient printing from your computer or for projection on a SMARTBoard™:

- Implementation Guide
- Math Stories
- Math Fun
- Student Response Book
- Assessment Manual



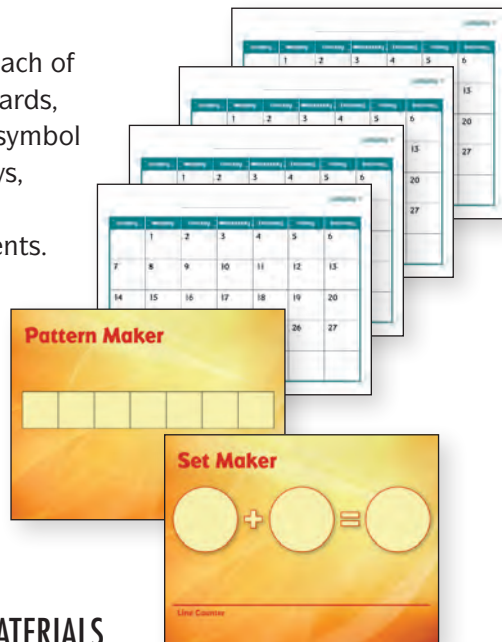
Lesson Manipulatives

Counting cubes, magnetic stars, rulers, paperclips, and theme-related counting objects are provided for every lesson. See Appendix B for a complete listing of the theme-related objects.



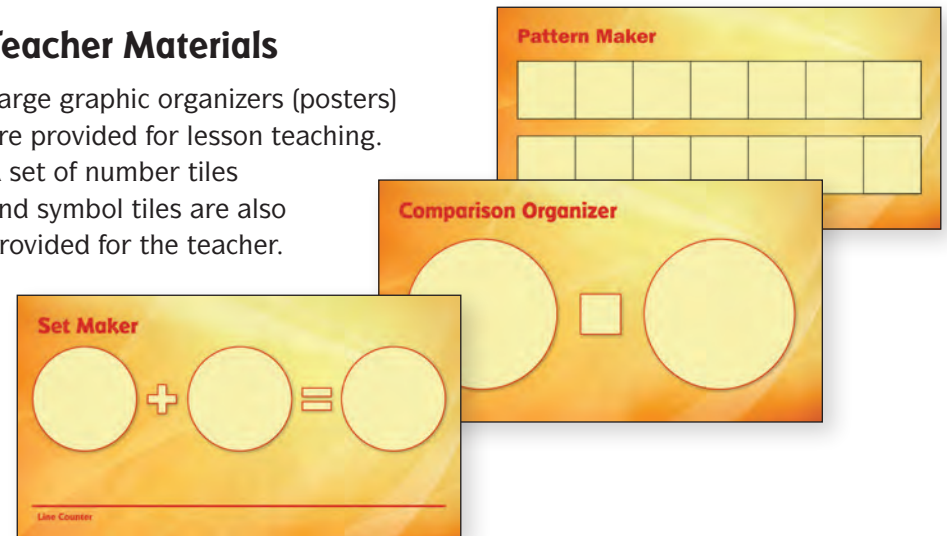
Student Materials

Early Numeracy includes 4 each of dry-erase/magnetic Work Boards, number lines, number tiles, symbol tiles, sets of calendar overlays, Pattern Maker overlays, and Set Maker overlays for students.



Teacher Materials

Large graphic organizers (posters) are provided for lesson teaching. A set of number tiles and symbol tiles are also provided for the teacher.



Games

For the review lessons, 4 game boards, 160 game cards, 4 game pawns, and a die are provided.



HOW TO USE EARLY NUMERACY

The **Early Numeracy** curriculum is designed to teach early numeracy skills to students with moderate-to-severe disabilities in a systematic manner. Students with moderate-to-severe disabilities often lack early numeracy skills altogether, lack fluency and automaticity with early numeracy skills, or have splinter skills. These deficits can interfere with a student's ability to access the general math curriculum at their grade levels to the maximum extent possible. The purpose of the Early Numeracy curriculum is to help build a strong foundation of these early numeracy skills, so students with moderate-to-severe disabilities are better prepared to work toward accessing the general curriculum. The Early Numeracy curriculum is not a precursor to teaching grade-aligned academics in mathematics, but should be used *concurrently* in order to strengthen students' numeracy skills while also providing opportunities to practice the skills in different contexts, environments, with different materials, and possibly with different instructors.

The Early Numeracy curriculum is designed to be taught in small groups of two to four students. (For information on how to group students, see p. 23.) Although the lessons can be taught individually, teaching in small groups is more practical and feasible. The curriculum is based on preliminary research done in actual schools.

Underlying Principles

The Early Numeracy curriculum expanded on the *Learning Trajectories for Young Children* (which were designed for typically developing students) by modifying them for students with moderate-to-severe disabilities and combining them with evidence-based practices for teaching mathematics to students with severe disabilities (Browder et al., 2008). To develop an effective method for teaching early numeracy skills in a highly structured manner, the Early Numeracy curriculum is based on the principles of direct and systematic instruction.

Direct instruction is a teaching model that facilitates development of skills in the most effective and efficient manner possible. It entails teaching students in small steps with practice after each step, leading students through the initial practice in an explicit and systematic manner, and continuously monitoring student progress to ensure that students have a high rate of success during practice (Przychozdzin, Marchand-Martella, Martella, & Azim, 2004). A direct instruction approach has been shown to be effective in teaching mathematics to students with disabilities, specifically those with learning disabilities, but now the effectiveness of this approach is branching out to students with other disabilities as well (Stein, Kinder, Silbert, & Carnine, 2005).

Systematic instruction also is based on the principles of applied behavior analysis (Wolery, Bailey, & Sugai, 1988) and entails identifying specific objectives to teach, planning procedures for teaching the objectives, implementing the procedures, monitoring student progress on an ongoing basis, and making instructional decisions based on the data. Browder et al. (2008) found that systematic instruction procedures, specifically constant time delay with feedback and the system of least intrusive prompts, are two evidence-based practices for teaching mathematics to students with moderate-to-severe disabilities.

Using Time Delay

- 1 What is time delay?** It's a method of systematic prompting and fading of a prompt using small increments of time.
- 2 Why does time delay work?** The rationale for using time delay as a method of instruction is that it suppresses errors.
- 3 Is there research to support the use of time delay?** The time-delay procedure is an evidenced-based practice (Jameson, McDonnell, Polychronis, & Ruesen, 2008; Spooner, Knight, Browder, Jimenez, & DiBiase, 2011).

Constant time delay is a procedure shown to be effective for students with moderate-to-severe disabilities (Ault, Wolery, Doyle, & Gast, 1989; Browder, Ahlgrim-Delzell, Spooner, Mims, & Baker, 2009). During this procedure, when first teaching a skill and giving a direction, the teacher points to the correct response immediately after giving the direction to the student. This is known as the zero time-delay interval. For example, the teacher lays out three cards with the operations “+,” “−,” and “x.” She gives the instructional direction, “Point to plus,” and immediately she also points to the card with the plus symbol so the student knows where to point. When the student consistently responds at zero time delay (referred to as Round 1), the teacher gives the direction to the student but delays prompting for a specified number of seconds (e.g., 4–5 seconds) to provide the student with the opportunity to respond independently. This is known as a 5-second delay interval. Teachers should know that the amount of time specified during the delay interval should be individualized for the student. For example, if the student has a physical limitation that requires more time to respond, the number of seconds should be increased to provide the student with enough time to respond. Table 1 presents the time-delay procedure applied to learning numerals. The procedure is also provided in Appendix A, where it can be printed and laminated and used as a reference guide during teaching.

Several questions should be considered for each student when using this procedure:

- What type of response do you want from the student: Point to the answer (receptive)? Say the answer (expressive)? Pull the answer from a choice board? Eye gaze to an answer?
- To respond, will the student use receptive only or receptive and expressive responding: Point to the answer only? Point to the answer and say it? Use an AAC device to respond?
- If the student requires a prompt, what type of model will you give (e.g., if the student is to say the number while pointing to it, model saying the number while pointing to it; if the student is to point only, model pointing)?

- How many warm-up trials will you give at 0-second time delay?
- How long will you wait before prompting the student for Round 2: 5 seconds? Or work progressively starting at 4 seconds, then 6 seconds, then 8, etc.?

Using Least Intrusive Prompts

The system of least intrusive prompts is another systematic instruction procedure shown to be effective for students with moderate-to-severe disabilities. This procedure uses a prompting hierarchy to guide students to make a correct response during instruction. First, the teacher provides the student with an opportunity to respond independently. If the student does not make a correct, independent response, or does not respond within a set period of time (e.g., 4–5 seconds), the teacher provides the next least intrusive prompt (e.g., a verbal cue). The teacher proceeds through a prompting hierarchy until the student elicits a correct response (Collins, 2007). As shown in Figure 1, this system places prompts given to students in a hierarchy from the least intrusive and most independent to the most intrusive and least independent. For specific information on how the system of least intrusive prompts is embedded in the Early Numeracy curriculum, see Table 3 on pp. 28–29.

The Early Numeracy curriculum combines the principles of direct and systematic instruction into the curriculum by embedding the system of least intrusive prompts and the constant time-delay procedure into the scripts.

Lesson Themes

Within each unit, themes are used to engage the students in learning the skills. These themes are listed in Table 2 on p. 14. Lesson themes vary and never repeat to help maintain interest throughout the curriculum. During each unit, students work on the same set of objectives, but by using different themes in each lesson, students have opportunities for repeated practice trials to mastery without losing interest in the lessons. This variation also promotes

TABLE 1 Time-Delay Procedure

ROUND 1: 0-Second Time Delay	ROUND 2: 5-Second Time Delay
<p>Round 1 is a warm-up round. A student (S) may need numerous trials at Round 1 before moving to Round 2.</p> <p>Step 1 Present the numeral and 3 distractor numerals. Review the numbers with the S.</p> <p>Step 2 In this first round, give the direction to find the number (e.g., say to the S, Find number 5). Provide an immediate prompt (0-second time delay) by pointing to the number while giving the direction.</p> <p>Step 3 Provide feedback.</p> <ul style="list-style-type: none"> ■ If the S points to the correct number, provide praise (e.g., Yes, you found 5). ■ If the S does not point to the correct response, use a physical prompt to help the S locate the correct number. Then give praise (e.g., Very good. You pointed to number 5). <p>Step 4 Repeat for the next numeral you are focusing on.</p> <p>Step 5 Repeat these steps for each S in the group.</p> <p><i>Note: There should be no errors in this round unless the S refuses to point or cannot imitate pointing. Do 0-second time delay two or three times. When the S consistently responds, move on to a 5-second time delay (Round 2).</i></p>	<p>Round 2 provides the S with the opportunity to respond independently. In Round 2, give the S up to 5 seconds (or whatever time you select) to respond before giving a prompt. If the S is unsuccessful on a 5-second time delay, return to Round 1 (0-second time delay).</p> <p>Step 1 Present the numeral and 3 distractor numerals to the S.</p> <p>Step 2 In this second round, give the direction to find the number (e.g., say to the S, Find number 5). Wait 4–5 seconds (5-second time delay) for the S to independently respond or to begin to initiate a response. Tell the S to wait if he or she isn't sure, If you are not sure, wait and I will show you.</p> <p>Step 3 Provide feedback.</p> <ul style="list-style-type: none"> ■ If the S points to the correct number, provide praise (e.g., Yes, you found 5). ■ If the S does not point to the correct response, use a physical prompt to help the S locate the correct number and say, for example, This is number 5. Then repeat the direction, Find number 5. <p>Step 4 Repeat for the next numeral you are focusing on.</p> <p>Step 5 Repeat these steps for each S in the group.</p>

FIGURE 1 System of Least Intrusive Prompts

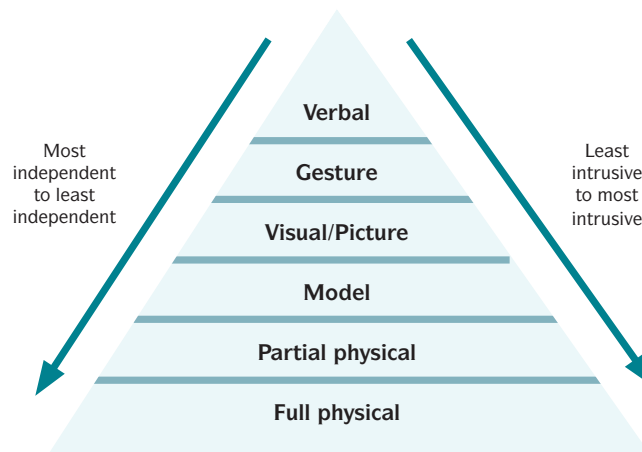


TABLE 2 Lesson Themes

UNIT ONE	Math Is Everywhere
Lesson 1	Math at the Speedway
Lesson 2	Math Treasures
Lesson 3	Gardening with Math
Lesson 4	Beach Math
Lesson 5	Math Class Trip
Lesson 6	Soccer Review
UNIT TWO	Math at Celebrations
Lesson 1	Mardi Gras Math
Lesson 2	Math in the New Year
Lesson 3	Math at the Fiesta
Lesson 4	Math at the Family Feast
Lesson 5	Going to a Pow Wow
Lesson 6	Basketball Review
UNIT THREE	Math in Nature
Lesson 1	Math in the Flower Garden
Lesson 2	Backyard Buggy Math
Lesson 3	Fishing for Numbers
Lesson 4	Math at the Aquarium
Lesson 5	Froggy Math
Lesson 6	Football Review
UNIT FOUR	Math + Me = Fun
Lesson 1	Butterfly Math
Lesson 2	Math at the Ballgame
Lesson 3	Math in the Desert
Lesson 4	Math at the State Fair
Lesson 5	Math in the Berry Patch
Lesson 6	Baseball Review

generalized responding across the content. In the field-test phase, teachers worked on one lesson for a week (at least four days) meaning each lesson is repeated 4 to 5 times. This pace may be either too slow or too fast for some students.

Lessons are designed to be culturally responsive, and some are designed to be informative of events/holidays or places throughout the United States that some students may never get to experience otherwise. Lessons with a cultural theme (e.g., Pow Wows) were reviewed for accuracy by an educator from that cultural group (e.g., Native American).

Objectives

The Scope and Sequence on pp. 7–8 lists objectives taught in each unit and lesson. The learning objectives reappear and become progressively more difficult as students progress through each unit. Some skills may be eliminated in higher units (e.g., number identification); whereas, other skills may be added in higher units (e.g., naming numerals). Each objective and the method of instruction were selected from current research. For more information on the learning objectives—including the rationale/goal, progression of skills, and method of instruction for each objective—see Table 3 on pp. 28–29.

Lesson Materials

Before you begin teaching, look through the materials included with the curriculum. Some materials will be used across all units (for example, for the teacher: Math Stories, Set Maker poster, Comparison poster; for the students: Work Boards with magnetic number lines, number tiles, symbol tiles, calendar overlays). These materials are pictured on pp. 9–10. In addition to these materials, there are specific manipulatives that accompany each lesson. These materials are included to support the theme of the lesson and to keep students engaged. An inventory of the manipulative theme-related materials is provided in Appendix B for your reference.

The first page of each lesson plan indicates exactly which materials are needed for each lesson. It is important to gather all materials needed for a lesson before teaching, so the lesson pace remains quick. Note that for ease in locating and organizing some items related to a unit, a color-coding system is used. The code is as follows:

Unit One  Unit Two  Unit Three  Unit Four 

This color code appears as a border on the pages of the Assessment Manual, the Student Response book, Math Fun pages, and game cards used for lesson review.

The Early Numeracy curriculum also comes with a CD containing PDFs of the Math Stories, Math Fun, and the Student Response Book. These PDFs can be projected onto a SMARTBoard™ for full group viewing, or can be used for printing individual pages for students. These files are also provided so adaptations can be made for individual students who may need materials enlarged or cut apart for eye gazing, etc.

Lesson Format

The Early Numeracy curriculum lessons follow the same format across all units. Each lesson is scripted to make it easier to teach, to minimize lesson preparation, and to ensure each lesson is taught consistently and systematically across all units. The teacher scripts are provided in color. For each lesson, follow the task analysis, which embeds least intrusive prompting. All 12 early numeracy skills/objectives are embedded within the lesson, so the students will get repeated practice over time and will build fluency. However, the order that these skills/objectives are presented varies from lesson to lesson. This prevents students from memorizing the answers using the lesson sequence.

■ **Anticipatory Set.** The first component of the lesson is to introduce the day's theme in an anticipatory set. Show students

the manipulatives for the lesson. Give students time to touch and view the materials before moving on in the lesson.

■ **Rote Counting Warm-up.** Review rote counting for the unit. Count with the students in chorus and then choose a student to count individually. Do this step quickly to encourage rote learning.

■ **Time Delay for Numeral Recognition.** Next, the lesson uses the time-delay procedure to teach the students to identify numerals 1–10. Students use their Work Boards with their number lines and number tiles adhered to it. In Round 1, name a numeral and model identifying the numeral (with zero delay or no delay) by holding up a number tile. Have the student find the numeral on their number lines at the same time. In Round 2, name a numeral but delay your prompt by 4–5 seconds to give the student the opportunity to *independently* respond. If the student does answer correctly without help, praise should be enthusiastic. If the student does not respond within 4–5 seconds, model the correct response, have the student follow your model, and praise the student for following your model. Errors should be discouraged because they can interfere with the pace of learning. If a student makes an error, remind the student, “If you are not sure, WAIT and I will help you. Don’t guess.” If students repeat errors, it would be an acceptable modification of the lesson to use the zero delay on both rounds and to try fading to 5 seconds the next day. In contrast, if students have begun to identify the numerals, save time by dropping the zero-delay round. A script for time delay is provided in Appendix A. Print and laminate for easy reference while you are teaching.

■ **Math Story.** The third component of the lesson is to read a math story. The stories are more than simple word problems. Instead, they create a theme to introduce multiple numeracy concepts. They also provide an opportunity for students to develop an awareness of “math” around them in their daily lives (e.g., 4 monkeys at the zoo). The story should be read aloud with interest and animation. Some students may be able to read the math story independently. Numeric text in the story can be changed to focus on different numbers (e.g., any of numbers 1–5 for Unit One). This text is

presented with a red font as a reminder that you should substitute other numbers you wish to focus on when repeating lessons.

■ **Application of Numeracy Objective to the Math Story.** After the story is read, work with the Early Numeracy materials provided for the lesson to demonstrate each math objective. Read parts of the story again (**highlighted with a colored background within the lesson**) but pause to integrate and teach the numeracy objective. These objectives are taught the same way each day using a system of least intrusive prompts. The order of the objectives is varied in lessons within a unit so students do not memorize a sequence of responding. All objectives include a chart to indicate how to use the system of least prompts. The cue is provided in the chart and well as the independent response to expect from the student. First, wait for the student to respond independently. Whenever a student correctly responds, provide praise. If the student responds incorrectly or doesn't respond, model the correct response and give the student another opportunity. Some students may need additional physical guidance to make the correct response. Procedures for assisting and correcting are also listed in the chart. To help fade this prompting, provide more enthusiastic praise when a student makes the response with less prompting than occurred on the prior day. Also, interrupt errors, and move to a less intrusive prompt if the student begins to respond incorrectly.

By following the script, shown in colored text, it will become easy across time to follow this prompt hierarchy. The hierarchy can be modified for students with sensory impairments. For students who are deaf, use a gesture instead of a verbal prompt. For students who have visual impairments, use additional verbal prompting prior to physical guidance (omitting the model). Note that red text in the lesson indicates to substitute the numbers you wish to focus on for the lesson.

■ **Math Fun.** At the end of each lesson, students practice specific objectives of the unit using a math activity page found in their Math Fun books. The activity pages are related to the theme of the lesson. Because lessons are repeated up to five times, three activity pages

are provided for each lesson, while additional activity pages can be printed from the CD if students need activity pages for the 4th and 5th days of teaching a lesson. Print more activity pages from the CD if you want students to have more opportunities to practice the skills. These thematic activities help the student associate the hands-on math experience with the symbols that represent the concept. Some students may be able to do these activity pages independently. Other students may need you to prompt each response on the activity page. Use the system of least intrusive prompts to help students complete the activity.

■ **Review Lesson.** The final lesson—Lesson 6—in each of the four units is a review lesson. The review lesson follows a different format. In these review lessons, the target is fluency for performing the math objectives. Instead of a math story, students receive quick trials for each math objective in a game format. They keep score of their correct answers using a game board and game cards. Correct independent responses earn more points than correct prompted responses. These review lessons help students gain the level of independence in responding that will be needed to show mastery during the end of unit math assessment.

How Often to Teach

The lessons are designed to be repeated over several days so students build fluency. Teachers in the pilot study repeated each lesson over the course of one week (4–5 days of instruction on each lesson). Lessons should be taught at a brisk pace with rapid opportunities for student responding. Without a rapid pace and active student responding, students will lose interest and motivation.

For some students, the lessons alone may not provide enough repeated exposure to master the content. For these students, it is acceptable to do mass practice during one-on-one instruction with you or a paraprofessional. For example, if a student is having difficulty mastering the calendar and measurement skills, or is

making slow progress, you may decide to conduct extra practice of these two skills during one-on-one instruction time with the student.

Monitoring Progress

Monitoring student progress is essential to the instruction of students with moderate and severe disabilities. It is very important to monitor student progress in each unit to see if the Early Numeracy curriculum is promoting learning and mastery for the targeted students. There are several ways to monitor student progress. First, a Progress Monitoring Form for each unit is provided so you can collect data on student responses during the lesson for each objective. These forms are provided in Appendix C and on the CD for convenient printing. Because the form is broken down objective by objective, it is easy to see on which skills students may need extra practice or one-on-one instruction.

Another way to monitor student progress is through the end of unit assessment. The Early Numeracy Assessment can be used to determine if students have reached mastery on unit objectives before moving on to the next unit of instruction. More information is provided on student assessment beginning on p. 25 and in the **Early Numeracy Assessment Manual**. The Early Numeracy Assessment Form is located in Appendix D and on the CD.

Finally, forms are also provided in Appendixes E, F, and G to plan for and record student responses during embedded instruction in a general education math class. Monitoring the student's progress in the general education math class is discussed in more detail on p. 18.

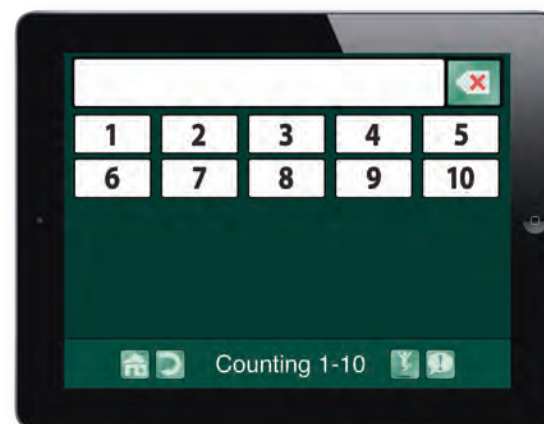
Adapting Materials

The materials for the Early Numeracy curriculum can be adapted for any student's individual response mode. The Student Response Book, Math Fun, and Math Stories are provided on the CD for convenient printing. Pages from these items can be enlarged, laminated, Brailled, cut-apart and attached to eye gaze boards (e.g., Plexiglas® boards), or used with augmentative/alternative devices.

Most of the Early Numeracy curriculum instructional cues only require a physical response, rather than a verbal response. For example, several of the instruction cues begin with, "point to," "touch," "make a set of," or "show me." For students with physical limitations this may be a challenge. Students with physical limitations may eye gaze to choose a response from an array of options. Also, students with physical limitations may count and/or indicate quantities using a variety of response modes, such as tapping, clapping, blinking, or nodding. For example, when a student who is nonverbal with a physical limitation is shown number "3" and asked, "What number is this?", he or she may respond by blinking three times to indicate the quantity that corresponds with the number.

Another challenge may be oral counting for students who are nonverbal. Skills that require this are rote counting and counting with one-to-one correspondence. AAC devices could be used for voice output and should be programmed with numbers 1–5 (Unit One), 1–10 (Unit Two), 1–15 (Unit Three), or 1–20 (Unit Four) to encourage verbal responses. Figure 2 shows an application that can easily be programmed and provides students with the opportunity to express early numeracy skills. For rote counting, the device could

FIGURE 2 Go-TalkNow iPad App



be programmed with the numbers spoken in sequential order and the nonverbal student may count “aloud” by touching one button that says the numbers in sequential order then touching the objects as the counting is spoken. Another method is to use a SMARTBoard™ with numbers that say their name when touched. When counting with one-to-one correspondence the student will: touch the first object, and then touch number 1 on the device; touch the second object, and then touch number 2 on the device; touch the third object, and then touch number 3 on the device; and so on, until the student has counted all of the objects. This is for a beginning student who is just learning to count with one-to-one correspondence. More advanced nonverbal students can simply count all the objects and then touch the number which represents the quantity. If technology is unavailable, provide the student with a number line, although this method will not provide the student with voice output.

For counting with one-to-one correspondence, there are other strategies too that address this skill. Some students who are nonverbal may be able to point to an item and wait for you to count (e.g., the student touches the first penny, you say “1”; the student touches the second penny, you say “2”; and so on). Students should count objects in order from left to right, and then move to the next row, if applicable. Although the verbal output is dependent on you, there are several signs that a student may not understand the concept of counting with one-to-one correspondence when using this strategy. For example, you continue to count at a methodical pace, and the student begins touching the objects off count or skips objects, then he or she is not counting with one-to-one correspondence. Never hurry counting aloud to keep up with the student’s finger. Also, if you count at a methodical pace aloud and suddenly pause, the student should indicate a pause in counting by not moving his or her finger or by looking up at you. The previous strategy described is teacher dependent.

When any changes are made to the instructional cues to fit a student’s response mode, the changes need to be consistently made throughout the scripted lessons. The changes also need to be made

during assessment procedures or inclusionary lessons in general education math classes.

Inclusion in General Education Math Class

This curriculum is intended to help students become fluent in early numeracy skills to better prepare students to participate in the general education math curriculum. During the pilot study, students participated in a general education math classroom with peers of their same chronological age to investigate if the skills learned generalized to new materials, activities, and math problem-solving activities. In the research phase, a paraprofessional was trained to conduct practice trials of each of the objectives during natural breaks in the classroom instruction or when the instruction supported embedding a skill. *Embedded instruction* is the strategy of providing opportunities for a student to learn or show mastery of a skill during the naturally occurring lesson. Typically, embedded opportunities are provided to the student and distributed over the course of an activity or lesson. Recently, the research on the use of embedded instruction supports it as an effective strategy to support the academic achievement of students with severe disabilities in inclusive settings (Carter, Sisco, Melekoglu, & Kurkowski, 2007; Jameson, McDonnell, Polychronis, & Reison, 2008; Jimenez, Browder, Spooner, & DiBiase, 2012). For example, if a 4th grade inclusive math lesson instructed students to find the area of a rectangle with the dimension 3" x 5", the paraprofessional would embed two opportunities of number identification for the students with severe disabilities, while other students computed the answer. In the pilot study, the paraprofessional embedded the skills using constant time delay, providing up to three embedded trials of all objectives from the unit of instruction currently being taught. A script for using constant time delay is provided in Appendix A.

Any time a student with moderate or severe disabilities is included within a general education math setting, collaboration between professionals is key to the student’s success. The principles of universal design for learning should be applied when developing

lessons and selecting modifications that ensure all students are included. For more information on UDL, visit the Center for Applied Special Technology website (CAST) at: <http://www.cast.org/udl/>

Several planning and data collection forms are provided in Appendixes E, F, and G to help with the collaboration and planning process for including students with moderate and severe disabilities in the general education math lesson:

■ **Appendix E: Embedded Instruction Planning Form** helps you, a paraprofessional, and a general education teacher plan how and where to embed numeracy objectives within the general education lesson.

■ **Appendix F: Inclusion Data Form I** helps you, a paraprofessional, and a general education teacher identify which objectives will be embedded and when within the lesson they will be embedded. This form provides a place to list more specific information on prompts to use and includes a place to collect data on the student's performance. One form per unit is provided.

■ **Appendix G: Inclusion Data Form II** helps the paraprofessional collect student performance data by unit and objective. It can be used to monitor an individual student or a small group of students. One form per unit is provided.

Each form and how to use it is explained in more detail next.

Embedded Instruction Planning Form. There are two components to planning. The Embedded Instruction Planning Form is a planning document for you, the paraprofessional, and the general education teacher to develop modifications to the grade-level lesson in order to help a student participate. This page lists five common components of a classroom lesson: teacher input/introduction, guided practice, group activity, independent practice/worksheet, and closure. The form provides a place to plan for specific modifications and accommodations to make within the lesson components. Planners should consider what is needed to help the student with moderate or severe disabilities participate in the general education lesson. Note that not every component of a lesson will need a modification

as there will be some activities that the student with disabilities can do without modifications.

A few examples of modifications would be:

- During guided practice—use numbers 1–10 only
- During the group activity—work with a peer buddy to solve a problem
- During independent practice—provide a calculator to solve problems

The general education teacher could also provide the group activity or independent practice/worksheet ahead of time so it can be modified or the problem simplified, if needed. Paraprofessionals may add pictures to support text in word problems or supplement the worksheet with graphic organizers to help students solve math problems. An example of a completed Embedded Instruction Planning Form is provided in Figure 3 on p. 20.

The final column of this form provides space for the paraprofessional to comment on what worked or did not work for that lesson. This feedback will be helpful for both the general education teacher, the paraprofessional, and/or a resource teacher when planning for future lessons. To be certain comments are not forgotten, paraprofessionals are encouraged to write comments throughout the lesson and/or as soon as the general education lesson ends.

In addition to the Embedded Instruction Planning Form, Inclusion Data Form I or Inclusion Data Form II should be used to plan more specifically which objectives will be embedded. For both forms, the objectives of the unit the student is currently working on during Early Numeracy instruction are listed in the first column. Both forms also include a place to take student data during the inclusive lesson. There are four response options to circle on this data sheet: 0, P, I, EC. The “0” indicates a 0-second delay was used because the student was still learning the skill and not responding independently yet (such as what may occur during the first two days of instruction). A “P” indicates that a student needed a prompt to respond. An “I” indicates the student independently answered correctly. There are

FIGURE 3 Example of Embedded Instruction Planning Form

Student Michaela Unit Two Date 11/12/13

Lesson components	Describe embedded instruction and how student will participate	Describe what worked well and what needs refining
1 Teacher Input/Introduction to Lesson		
2 Guided Practice		
3 Group Activity	Michaela will participate within a group. Assign peer buddy for writing assistance.	
4 Independent Practice/ Worksheet	Use same worksheet as entire class, but Michaela will solve problems using numbers 1-5. Michaela can also use a calculator for the worksheet problem.	
5 Closure		
Comments:		

several instances when this could happen, such as after waiting and asking the teacher for help, or after not responding within a designated period of time. An “EC” indicates a student needed an error correction, such as after pointing to an incorrect response. It is very important when performing an error correction to remind students to wait if they are not sure, “Remember, if you are not sure, wait, and I will help you.” One rule of thumb for implementing

the constant time delay in the general education classroom is that if a student gets all “P” or “EC” on an objective for two consecutive sessions/days, the paraprofessional should go back to using a 0-second delay for at least one session/day. Constant time delay is an errorless learning procedure, and the goal is for students to make only correct responses and not guess.

Inclusion Data Form I includes a place to plan when to embed the skills during the lesson, and a place to list the specific prompt to use if the student makes an error when responding, or to note that he or she does not respond within a designated period of time. Prompts could follow the time-delay procedure or a least intrusive prompt hierarchy. Figure 4 is an example of a completed Inclusion Data Form I for Unit One.

All three forms are structured to help the inclusion process flow as smoothly as possible. During the pilot study, paraprofessionals enjoyed using the planning documents, and this helped ease fears and make inclusion an enjoyable experience for all. The paraprofessionals continued to use the forms throughout the course of the study to plan with the general education teacher; however, as the paraprofessionals became more familiar with embedding the

FIGURE 4 Example of Inclusion Data Form I: Unit One

Student Julio Date 4/15/13

Directions: Identify at least six numeracy skills per lesson to embed, when to embed them, and prompts to use. Track student performance using this code:

O = 0-second delay **P** = Prompt given after 4–5 seconds **I** = Independent, correct response within 5 seconds **EC** = Error correction

Objectives	Best Time to Embed	Prompts	Student Performance
<input type="checkbox"/> 1 Count 1–5 movable objects in a line.	Guided practice p. 67 count blocks representing numbers in the problem	I say, <i>Count like this: 1, 2, 3.</i> Move each item off the line as I count. Count them all and then I say, <i>Your turn to count.</i>	O P I EC
<input type="checkbox"/> 2 Count 1–5 nonmovable objects in a line.	Same	I say, <i>Count like this: 1, 2, 3.</i> Touch each item as I count. Count them all and then say, <i>Your turn to count.</i>	O P I EC
<input type="checkbox"/> 3 Rote count from 1–5.	During homework review	I say, <i>1, 2, 3.</i> Activate the AAC device to model. Then I say, <i>Now you say it.</i>	O P I EC
<input type="checkbox"/> 4 Make sets of 1–3.	p. 67	I model first, <i>Put four in the circle like this: 1, 2, 3, 4.</i> Now you try.	O P I EC
<input type="checkbox"/> 5 Add premade sets with sums to 5.	Guided practice Add sets corresponding to problems on p. 67	I model pushing counters to the last circle and counting them. I say, <i>Let's count to add like this: 1, 2, 3, 4, 5.</i> Now you try. I won't say plus; I'll just count the items.	O P I EC

objectives naturally, they relied less on the Inclusion Data Form I and transitioned to using Inclusion Data Form II, where they could record multiple students' responses on the same form.

Keys to Successful Inclusion

In addition to planning and collaborating with the general education teacher, there are some other considerations for successfully including students with moderate-to-severe disabilities in the general education classroom.

- Be mindful of how many students with disabilities to include in the general education math classroom. Avoid overloading an already full math classroom. A good rule of thumb is to keep a 2:1 or 3:1 ratio of students with moderate-to-severe disabilities to paraprofessionals.
- Ensure that students with disabilities are dispersed throughout the entire classroom, and that they have their own desks. Avoid putting all students with disabilities in one area or at a table in the back of the room with the paraprofessional. Although this is considered physical inclusion, the isolation inhibits students with disabilities from being meaningfully included and considered a full member in the general education classroom.
- Consider using peer supports. Seating students next to assigned peer buddies can help alleviate some of the overdependence on the

paraprofessional and general education teacher. Peer supports are great to use because the peers can help students with moderate-to-severe disabilities stay on-task, re-direct inappropriate behaviors, embed trials of math skills if properly taught to do so, and help students with disabilities feel welcomed in the classroom.

- Finally, be mindful of students' problem behaviors and address these before going into the general education classroom. Three strategies for improving problem behaviors are social stories, video modeling, and role playing. Not only can these strategies help improve inappropriate behaviors, but they can also be used to teach classroom rules, procedures, and routines prior to going into the general education classroom. In order to be successful, students must be prepared for the transition. Consult with the general education teacher and find out what classroom procedures and routines are most important in his or her classroom. Common classroom procedures and routines may include things like entering the classroom and finding a seat, using the restroom, getting the teacher's attention, and getting materials in the classroom or turning in assignments. Students with severe disabilities should be held to the same expectations as students without disabilities in the classroom. This will also help them be considered full members of the inclusive setting.

EARLY NUMERACY ASSESSMENT

The Early Numeracy assessment is provided for two purposes:

- 1 To help you determine where to start instruction.
- 2 To help determine mastery of unit objectives and thus determine when a student is ready to move on to the next unit.

How to Conduct the Assessment

To conduct the assessment, have all materials readily available before beginning a testing session with a student. Also have available an Early Numeracy Assessment Form (Appendix D). Other materials needed for the assessment are listed in the **Early Numeracy Assessment** manual. It is prudent to practice administering the assessment with another teacher or paraprofessional before testing a student in order to become comfortable with the assessment. Maintaining a fast pace is crucial to keeping a student's interest and motivation. The following are rules for administering the Early Numeracy assessment:

- Set up the Early Numeracy Assessment manual so directions face you and the student page faces the student.
- In the assessment, with the exception of skill 7, each skill includes a demonstration item. **During the demonstration, you may prompt the student to be certain he or she understands the task.**
- Most skills have two test items. **When administering the test items, do not prompt or provide any error correction.** Student responses should be made independently.
- Reinforce the student for his or her performance during test items and not for correct responses. Examples of verbal reinforcement for performance are: “Great job!” “I like the way you answered that question!” “You are working so hard, keep it up!”
- Score test items as + for correct, independent responses, and – for incorrect responses or no response.

- Using the Early Numeracy Assessment Form, total the items correct, and calculate a percentage correct.
- **The assessment should last no longer than 20 minutes with a student.** If the assessment is taking longer, consider practicing administering the assessment a few more times until you become fluent. For more information on these, see the FAQ section on p. 25.

Determining Where to Start Instruction

Not all students need to begin at Unit One in the Early Numeracy curriculum, but you should not guess where to begin instruction. Using the Early Numeracy Assessment, you can use data to make the decision. To determine where to start instruction, begin testing at Unit One. If the student scores 80% or higher, administer Unit Two, and so forth until the student scores less than 80%. Start instruction at that unit. Figure 5 on p. 24 presents an example.

In this example, the percent correct for Unit One was 85%, consequently Unit Two was administered the next day. The student scored 63% correct on Unit Two. The student scored more than 80% correct on Unit One, but less than 80% on Unit Two, so the teacher began instruction with Unit Two. Note that Jon had a splinter skill of rote counting to 20 but lacked knowledge of number concepts, so Unit Two was an appropriate starting point.

Grouping Students by Level

Once you have determined where to start each student, students in the classroom can be grouped by level. For example, you can group two students who are starting in Unit One and form another group of three students who are starting in Unit Three. It is best to not have more than three students in a group at a time to make sure

FIGURE 5 Example of Early Numeracy Assessment Form

Student Jon Examiner Rosario Williams

School Vernon Elementary

Student Response Mode Modifications

☒ AAC device

☐ Eye gaze

☒ Other (explain) Needs 5-sec. wait time.

Unit Scores

Unit One Score 85% Date 9/15

Unit Two Score 63% Date 9/16

Unit Three Score _____ Date _____

Unit Four Score _____ Date _____

Directions: Circle + for a correct, independent response or – for an incorrect response or no response.

Domain	Unit One Skills	Item	Score	Unit Two Skills	Item	Score	Unit Three Skills	Item	Score	Unit Four Skills	Item	Score
Counting	1 Count 1–5 movable objects in a line.	1	⊕ –	1 Count out 1–5 movable objects from a group.	1	⊕ –	1 Count 1–10 movable objects in a line.	1	+ –	1 Count out 1–10 movable objects from a group.	1	+ –
		2	⊕ –		2	⊕ –		2	+ –		2	+ –
	2 Count 1–5 nonmovable objects in a line.	3	+ ⊖	2 Count 1–5 scattered, nonmovable objects.	3	+ ⊖	2 Count 1–10 nonmovable objects in a line.	3	+ –	2 Count 1–10 scattered, nonmovable objects.	3	+ –
		4	+ ⊖		4	+ ⊖		4	+ –		4	+ –
	3 Rote count from 1–5.	5	⊕ –	3 Rote count from 1–10.	5	⊕ –	3 Rote count from 1–15.	5	⊕ –	3 Rote count from 1–20.	5	⊕ –
	Calendar	10 Identify dates from 1st to 5th on a calendar.	18	+ ⊖	10 Identify dates from 1st to 10th on a calendar. <i>Not 5 or higher</i>	18	+ ⊖	10 Name dates from 1st to 5th on a calendar.	18	+ –	10 Name dates from 1st to 10th on a calendar. <i>(Note: Accept ordinal or cardinal forms.)</i>	20
19			+ ⊖	19		+ ⊖	19		+ –	21		+ –
11 Identify 1–5 days later in a week using a calendar.		20	⊕ –	11 Identify 1–5 days later across 2 weeks using a calendar.	20	⊕ –	11 Identify 1–10 days later across 2 weeks using a calendar.	20	+ –	11 Identify 1–10 days later across 3 weeks using a calendar.	22	+ –
		21	⊕ –		21	⊕ –		21	+ –		23	+ –
Score	Unit One items correct: <u>22</u> / 26			Unit Two items correct: <u>19</u> / 30			Unit Three items correct: ____ / 26			Unit Four items correct: ____ / 32		
	Percentage: <u>85%</u>			Percentage: <u>63%</u>			Percentage: _____			Percentage: _____		
Comments: Jon spontaneously counted to 20, but one-to-one correspondence for counting 5–10 is lacking. Begin teaching at Unit Two.												

instruction is fast paced and to prevent too much wait time among students. This will also make data collection easier.

Using Early Numeracy Assessment to Determine Unit Mastery

To determine unit mastery, begin testing the unit just instructed. The suggested mastery criterion for units is 80% correct. You may make the mastery criteria more stringent for specific students. For example, if a student picks up on material quickly, you may choose to use a more stringent mastery criterion, such as 100% correct. However, students must meet the minimum mastery criterion of 80% correct to move to the next unit of instruction. If the student scores 80% or higher, he or she is ready to move to the next unit. If the student scores below 80%, several options exist:

- Repeat the unit and retest.
- Analyze the data more closely. Scan the assessment form to see if one or two specific objectives are keeping the student from moving on (e.g., the student gets both trials incorrect within a specific objective). If this is occurring, provide mass trials through one-on-one instruction for just those objectives, but move the student on to the next unit.

Assessment FAQs

■ How should I teach skills that my student didn't master in the previous lesson?

If the student did meet criteria (80%) to move onto the next unit of instruction, but didn't master all of the skills (e.g., identify numbers 1–5), plan to continue providing the student opportunities to master those skills. While teaching the next unit, you may find it useful to embed trials of the previous unit's objective into the lesson (e.g., while teaching students to identify numbers 6–10, embed trials to identify numbers 1–5). Also, provide massed trial instruction with the student on the particular objective from

the prior unit that was not mastered. Due to the nature of the curriculum, math skills build upon each other as the students progress from one unit to the next. The spiraling of the curriculum will automatically provide students an opportunity to maintain and continue to build skills from previous units.

■ What if when using the Progress Monitoring Form, I can see that a student has reached mastery criterion before the end of the unit?

There are several instructional options. First, you may choose to use a more stringent mastery criterion for this student. This will ensure the student knows the content with consistency and this also builds fluency. Second, you may choose to move to the next unit of instruction with that particular student without finishing all lessons within a unit. Third, you may choose to finish the unit with the student to build generalization and maintenance. This would likely be the case if the student met the mastery criterion early, but other students in the group had not yet met mastery criterion.

■ What if the student exhibits problem behaviors during the assessment?

First, make sure your administration is very fast paced. If you are not fluent with administering the assessment and this is causing excess wait time for the student, practice with another adult until you can administer the assessment without looking at the directions. Second, consider using some type of reinforcement with the student for working (e.g., token economy system). This should be student specific. Remember, a reinforcer is something that increases the likelihood of the behavior occurring in the future, so if you are providing a reinforcer to the student and his or her behavior is not improving, the reinforcer is not motivating and is not acting as a true reinforcer. Perhaps even have the student choose something that is motivating to work for to ensure it is a true reinforcer. This can also be done using a "first/then" approach (i.e., the Premack Principle). Again, the student chooses what he or she is working for to insert into the "then" chart or statement, "First work, then computer for 5 minutes."

A row of 12 teal squares, each 10x10 pixels, used as a decorative element.

26

This list is not exhaustive because the term number sense is defined differently by many experts and may include different skills. The plethora of definitions for number sense and the vagueness of these definitions can create potential problems. For example, many teachers are not familiar with number sense concepts and do not know how to teach them. Teachers also may not know how to introduce the skills in a sequential order. Learning trajectories have been developed to ameliorate these problems.

Learning trajectories help clearly establish big ideas in mathematical education for young children and lay out a path for educators to use to help students learn (Bowman, Donovan, & Burns, 2001; Clements, 2004; Fuson, 2004; Griffin, Malone, & Kame'enui, 1995; Sarama & Clements, 2009). From a national standpoint, learning trajectories are viewed as important because they facilitate instruction and learning based on developmental principles for all children. They are comprised of three components:

- 1** A clear goal of what students should learn
- 2** A developmental progression in which students move through levels of things
- 3** Instruction that leads students through this progression to attain the goal

Sarama and Clements (2009) developed learning trajectories for young children out of a four-year project funded by the National Science Foundation, which creates and evaluates math curricula for young children based on sound research and theoretical framework. Their findings are reported in the book *Early Childhood Mathematics Education Research: Learning Trajectories*. This book served as the inspiration for the development of the Early Numeracy curriculum; however, new learning trajectories had to be developed to address the needs and learning styles of students with severe disabilities.

Learning trajectories for the early numeracy skills included in the Early Numeracy curriculum were developed by elementary math and special education experts in severe disabilities (Drew Polly, Bree Jimenez, & Diane Browder). As a starting point for the learning trajectories, the experts compared where typically developing early elementary students would be in regards to their early numeracy skills with what a large majority of students with moderate and severe disabilities in elementary grades typically achieve. Then these experts reviewed three published curricula designed for children with disabilities and examined the specific skills—which aligned specifically with early numeracy skills—taught within these curricula. Finally, the experts considered the developmental levels and cognitive tools that students with moderate-to-severe disabilities were likely to have, as well as methods of instruction that are evidence-based practices for teaching mathematics to this population. Once all this information was gathered, the experts prioritized and selected skills based on ones they thought were necessary for students with moderate-to-severe disabilities to have in order to access grade-level content with the greatest success possible and for functional academic purposes. These prioritized skills became the learning goals for the Early Numeracy curriculum. Next the experts broke them down further into very discrete skills based on developmental progressions of children with moderate-to-severe disabilities. The targeted skills for Early Numeracy are listed in Table 3 on p. 28.

TABLE 3 Early Numeracy Goals and Skill Progression

Goal Area	Rationale for Goal	Progression of Skills in Early Numeracy	Instructional Method
Counting with one-to-one correspondence	Counting objects with one-to-one correspondence may be arguably the most important early numeracy skill. Students must have this skill to begin subitizing and to build on higher skills, such as place value (Sarama & Clements, 2009).	Higher numbers are introduced in later levels (e.g., 1–5 objects are introduced in Units One and Two; 1–10 objects in Units Three and Four). Also, the skill progresses from movable and nonmovable objects in a line, to scattered objects, to counting objects from a group where more objects are provided than needed.	Least Intrusive Prompts
Numerical identification	Students must recognize that numbers have numerical representations and verbal names (Sarama & Clements, 2009).	Higher numbers are introduced in later levels. For example, in Unit One numerals 1–5 are introduced and in Unit Two, numerals 1–10 are focused on. Students must identify numbers expressively in Units Three and Four.	Time Delay with Feedback
Rote counting	Students must know the verbal sequence of numbers in order to develop number sense (Sarama & Clements, 2009).	Higher numbers are introduced in later levels. For example, in Unit Two, students rote count 1–10; in Unit Three, students rote count 1–15; and in Unit Four, students rote count 1–20.	Least Intrusive Prompts
Composing sets	Students must understand the effect of combining objects to create sets (Sarama & Clements, 2009).	Students are required to create larger sets in later levels. In the highest level, students must create sets in the context of a story.	Least Intrusive Prompts
Addition with sets	Students must understand the effect of combining, and eventually decomposing sets of objects. Also, this is a prerequisite skill to understanding the meaning of operation symbols (+, −, ×, =) (Sarama & Clements, 2009).	Unit One uses premade sets, with sums of 5 or less. In Units Two and Three, students create sets and add sums of 5 or less and 10 or less, respectively. In Unit Four, students create sets with sums of 10 or less in context (word problems are introduced).	Least Intrusive Prompts

TABLE 3 *(continued)*

Goal Area	Rationale for Goal	Progression of Skills in Early Numeracy	Instructional Method
Symbol use and comparing sets	Students must develop the ability to discriminate and construct equivalent relations between sets using spatial magnitude (recognizing relations between size) and numerical magnitude (recognizing relations by counting) (Sarama & Clements, 2009).	Students compare sets of objects or pictures, progressing from identifying equal sets in Unit One, to the greater set in Unit Two, to the set with less in Unit Three. Students identify the correct symbol to use when comparing sets.	Least Intrusive Prompts
Patterning	Recognition and analysis of patterns is a prerequisite skill to algebraic thinking (Sarama & Clements, 2009).	Students progress from identifying an ABAB pattern, to extending an ABAB pattern, to creating ABAB patterns, and then determining the missing components of patterns.	Least Intrusive Prompts
Standard and nonstandard measurement	Students must understand the relationship between counting and measurement, which requires counting objects of continuous quantity. Measuring length requires students to identify a unit to measure with, to subdivide the object by the units mentally and physically, and to iterate, or place the units end-to-end alongside the object to measure (Sarama & Clements, 2009).	In Unit One, students begin using nonstandard units of measurement (e.g., paperclips) to measure. In Units Two and Three, students use a ruler to measure in inches, using higher numbers in Unit Three. In Unit Four, students convert inches to feet (for 1–3 feet only).	Least Intrusive Prompts
Calendar skills	Calendar skills require students to identify numbers and count forward with one-to-one correspondence. This is an example of an early numeracy skill that is applied using functional academics (review of math curricula written for students with disabilities).	Students begin by locating a specified date and moving across the week. Higher levels introduce finding dates with higher numbers, moving across 2 and 3 weeks on the calendar, and naming dates.	Least Intrusive Prompts

RESEARCH ON THE EARLY NUMERACY CURRICULUM ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■

The **Early Numeracy** curriculum was developed through Project MASTERY—which stands for **math and science teaching** that promotes clear **expectations** and **real learning across years**—for students with significant cognitive disabilities. Project Mastery is housed at the University of North Carolina, Charlotte, and is evaluated in the Charlotte-Mecklenburg School System. This project was funded by the Institute of Education Sciences (IES) Special Education Research Grant # R324A080014 awarded to the University of North Carolina, Charlotte. The principal investigator (PI) was Dr. Diane Browder and co-PI was Dr. Fred Spooner.

The Early Numeracy curriculum was built on research and was field-tested; however, more research is needed to extend the evidence base. Seven students with severe disabilities, including students with autism and moderate-to-severe intellectual disabilities received daily math instruction using the Early Numeracy curriculum, and participated in general education math classes with peers of their same chronological ages. The grant staff trained three special education teachers to implement with fidelity the Early Numeracy curriculum in their self-contained classrooms. In addition, the grant staff trained three special education paraprofessionals to embed instruction with fidelity in the general education classroom.

The researchers examined the performance of the students who participated in the pilot study (Browder et al., 2012). It is important to note that Unit Four was not field-tested due to time constraints and the school year ending. Results showed that all students acquired targeted early numeracy skills across units. Generalization of skills was observed across units prior to instruction for each student. This is likely due to the fact that there was carryover in the skills from unit to unit because the skills were broken down into learning objectives based on developmental progressions. For example, in Unit One, students made a set of 3 objects, in Unit Two, a set of 4 objects, and in Unit Three, a set of 9 objects. However, once students mastered learning to count with one-to-one correspondence and began subitizing (seeing how many

without counting), they were able to do so with any number of objects. Overall, generalization was only observed in the numbers and operations skills and patterning, but was not observed in the measurement skills. One interesting finding was that students were better able to perform the skills within the general education math setting when the trials were embedded within the daily instruction versus during the assessment which was given by grant staff in the self-contained classroom. This was likely due to the length of the assessment and satiation from being assessed weekly. The findings from the field test were very promising.

Additionally, Jimenez and Kemmery (2012) conducted a single-subject multiple probe across classrooms study that investigated the effect on early numeracy skills on five students (within three classrooms) when using the Early Numeracy curriculum. Three teachers of students with significant disabilities used the Early Numeracy curriculum to teach five elementary students (ages 7–11) early math skills. During baseline, all five students had limited early numeracy skills (ranging from 4.2%–34% mastery, with a mean of 17.8%). After teaching Unit One of the Early Numeracy curriculum, all five students significantly increased their early numeracy skills (ranging from 13.3% to 44% mastery, with a mean of 28%). Two of the three classrooms ($n = 3$) received training in Unit Two of the curriculum; the other class ran out of time due to the end of the school year. After teaching Unit Two of the curriculum, all three students significantly increased their early numeracy skills (ranging from 30–54% mastery, with a mean of 46%). Due to the end of the school year, Units Three and Four were not assessed. While data from this study only showed mastery at 46% of the total skills, it is important to note that only 48% of the total early numeracy skills within Units One and Two were taught. All five students showed a significant increase in early numeracy skills in spite of the brief period of instruction (2–3 months) with this curriculum (Jimenez & Kemmery).